MATHEMATICS 280

MULTIVARIATE CALCULUS

I. Introduction

A. Catalog Description

This course, a continuation of the calculus sequence that starts with MATH 180 and 181, is an introduction to the study of functions that have several variable inputs and/or outputs. The central ideas involving these functions are explored from the symbolic, the graphic, and the numeric points of view. Visualization and approximation, as well as local linearity continue as key themes in the course. Topics include vectors and the basic analytic geometry of three-space; the differential calculus of scalar-input, vector-output functions; the geometry of curves and surfaces; and the differential and integral calculus of vector-input, scalar-output functions. Computer software and graphing calculators are used to increase the range of problems which students can analyze. Students who have received credit for MATH 221 may not receive credit for MATH 280. *Prerequisite: MATH 181 or its equivalent.* Satisfies the Mathematical Approaches core requirement.

B. Objectives

The primary objective for students in this course is to appreciate the power and beauty of the calculus. In this multi-variable setting, students will begin to appreciate the central role of linearity. They will also see how approximation by polynomial functions is fundamental for understanding optimization problems. In the study of vector-input, vector-output functions, students will see how abstract concepts lead to a deeper understanding of important mathematical ideas. In particular, they will begin to experience the organic and highly interconnected nature of mathematics by using calculus to analyze and solve problems from the sciences and business related fields.

This course satisfies the Mathematical Approaches category of the university's core curriculum by developing an appreciation of the power of mathematics and formal methods to provide a way of understanding a problem unambiguously, describing its relation to other problems, and specifying clearly an approach to its solution. A student in this course will develop a variety of mathematical skills, an understanding of formal reasoning, and a facility with applications. Specifically, this course will expose students to formal logic to the extent that it is required to understand mathematical proof.

C. Prerequisites - Math 122 or its equivalent.

II. Required Topics

A. Vectors and analytic geometry of three-space

1. Component and geometric views of vectors
2. Vector algebra
3. Dot and cross products
4. Planes and lines in $\mathbf{R}^3$

II. Required Topics (cont.)

B. Differential calculus of scalar-input, vector-output functions

1. Visualizing scalar-input, vector-output functions
2. Limit, continuity and derivative
3. Arc length

C. Differential calculus of vector-input, scalar-output functions

1. Visualizing vector-input, scalar-output functions: slices and level curves
2. Limits and continuity
3. Partial derivatives, gradient, and directional derivatives
4. Higher order partial derivatives and Taylor polynomial approximations
5. Extrema problems: unconstrained and constrained

D. Integral calculus of vector-input, scalar-output functions

1. Double integrals and iterated integrals
2. Double integrals in polar coordinates
3. Triple integrals and iterated integrals
4. Triple integrals in cylindrical and spherical coordinates

E. Calculus of vector-input, vector-output functions

1. Curl and divergence
2. Line integrals
3. Surface integrals
4. Fundamental Theorems of Calculus: Stokes', Green's, Gauss'.

III. Bibliography

M.J. Strauss, G. L. Bradley, and K. J. Smith, *Calculus*

A. Ostebee and P. Zorn, *Calculus: From Graphical, Numerical, and Symbolic Points of View*

D. Hughes-Hallett et al., *Calculus*

J. Stewart, *Calculus: Early Transcendentals*

J. Marsden, A. Tromba, and A Weinstein, *Basic Multivariable Calculus*