MATHEMATICS 375
PROBABILITY THEORY AND ITS APPLICATIONS

I. Introduction

A. Catalog Description

This course provides an introduction to the standard topics of probability theory, including probability spaces, random variables and expectations, discrete and continuous distributions, generating functions, independence and dependence, special probability models, sampling distributions, laws of large numbers, and the central limit theorem. The computer is used as a tool to enhance one's understanding of randomness and the above mentioned concepts through simulation, and to solve difficult analytical problems numerically. An emphasis on modeling real-world phenomena is always present. *Prerequisites: Math 221, 232 or permission of the instructor*. Satisfies the proof-based requirement in major contracts.

B. Objectives

The primary objective is to provide students with both the general principles and specific details of probability and stochastic models. Special emphasis is given to those aspects of probability theory which form the foundations for the mathematics of statistical inference. Students completing Math 375 and 376 should be prepared to study for the first actuarial examination (Course 1).

C. Prerequisites

Mathematics 221, 232 or consent of the instructor.

II. Required Topics

A. Probability Spaces

1. Basic Set Theory
2. σ-fields
3. Probability Measures
4. Conditional Probability and Independence
5. Basic Combinatorics

B. Random Variables and their Distributions

1. General Concepts
2. Discrete random variables and random vectors
3. Continuous random variables and random vectors

C. Distribution Functions and Probability Densities

1. Discrete case
2. Continuous case

D. Moments, Expected Values and Inequalities

1. Moments of random variables
2. Expectations and Variance
3. Conditional Expectation
4. Probability and Moment Inequalities

E. Characteristic Functions and Moment Generating Functions

1. Definitions and Basic Results
2. Results for special distributions

F. Stochastic Independence

G. Basic Limit Laws

1. Modes of Convergence
2. Laws of Large Numbers
3. Central Limit Theorems

H. Transformations of Random Variables and Random Vectors

III. Optional Topics

A. Reliability Theory
B. Queueing Theory
C. Markov Chains
D. Birth and Death Processes

IV. Bibliography

Feller, Wm. An Introduction to Probability Theory & its Applications I & II, John Wiley & Sons

De Groot, M. Probability & Statistics, Addison-Wesley

Derman, C., L. Gleser, I. Olkin A Guide to Probability Theory and Application, HRW

Hoel, P., S. Port., C. Stone, Probability Theory, Houghton-Mifflin

Hogg, R. & Tanis, E. Probability and Statistical Inference, Prentice-Hall

Larsen, R. & M. Marx An Introduction to Probability & its Applications, Prentice-Hall

Parzan, E. Modern Probability Theory and its Applications, John Wiley & Sons

Rice, J. Mathematical Statistics and Data Analysis, Wadsworth

Roussas, G. Mathematical Statistics, Addison-Wesley

IV. Assessment

Student learning is assessed through the use of homework problems, in-class examinations, take-home examinations requiring detailed written solutions, and computer simulation exercises.