

## MATHEMATICS 335/COMPUTER SCIENCE 335 OPTIMIZATION

### I. Introduction

#### A. Catalog Description

An introduction to the principal areas of optimization--linear programming, mathematical optimization, and combinatorial optimization. *Prerequisites: Math 221, 232, CSci 161.*

#### B. Objectives

This course should introduce the student to three of the main branches of optimization--linear, nonlinear and combinatorial. Since each of these topics could constitute an entire course in its own right, by necessity this course should limit itself to presenting the main definitions and algorithms of each area, along with some representative applications. The students should also be made aware of the possibilities for further study in each area.

Together with course in mathematical modelling, probability and statistics a student can obtain a solid preparation for a career, or graduate study, in operations research. This course would also be especially beneficial for students studying economics, engineering or actuarial science.

#### C. Prerequisites

Because of the many problems in this course that are stated using multivariate functions and matrix notation, Math 221 and Math 232 are prerequisites. Because many of the algorithms developed would be implemented as computer programs, and to develop an appreciation of problems involving computational accuracy, Computer Science 161 is also a prerequisite.

### II. Required Topics

#### A. Linear Programming

1. Definition and applicability of a linear program
2. Simplex method, tableaux
3. Duality
4. Sensitivity analysis
5. Software packages for LP
6. Transshipment problems
7. Assignment problems
8. Discussion of goal programming, integer programming, quadratic programming
9. Discussion of interior point methods

#### B. Mathematical Optimization, Nonlinear Programming

1. Iterative methods for unconstrained optimization
2. Least squares optimization
3. Convexity, Karush-Kuhn-Tucker conditions
4. Methods for constrained optimization
5. Penalty functions

C. Combinatorial Optimization

1. Integer programming
2. Spanning trees
3. Shortest paths
4. Maximum flows
5. Minimum-cost flows
6. Assignments, matchings
7. Traveling salesman problem
8. Heuristics: genetic algorithms, simulated annealing, tabu search
9. Approximation algorithms

III. Bibliography

**Linear Programming** The text by Chvatal is a popular choice as a textbook. Williams has an abundance of detailed examples. Ignizio discusses interior point methods.

Brickman, Louis, *Mathematical introduction to linear programming and game theory*, Undergraduate texts in mathematics series, Springer-Verlag, 1989.

Chvatal, Vasek, *Linear programming*, W.H. Freeman, 1983.

Hartley, Roger, *Linear and nonlinear programming : an introduction to linear methods in mathematical programming*, Wiley, 1985.

Ignizio, J.P. & Cavalier, T.M., *Linear Programming*, Prentice Hall, 1994.

Luenberger, David G., *Linear and nonlinear programming*, Addison-Wesley, 1984.

Murty, Katta G., *Linear and combinatorial programming*, Wiley, 1976

Strayer, James K., *Linear programming and its applications*, Undergraduate texts in mathematics series, Springer-Verlag, 1989.

Williams, H.P., *Model Building in Mathematical Programming*, Wiley, 1993.

**Mathematical Optimization, Nonlinear Programming** Nemhauser is a comprehensive reference on many aspects of optimization in general. Peressini could be used as a textbook. The electronic textbook can be accessed via the WWW page mentioned earlier.

Franklin, Joel N., *Methods of mathematical economics : linear and nonlinear programming : fixed-point theorems*, Undergraduate texts in mathematics series, Springer-Verlag, 1980.

Nash, S. G. & A. Sofer, *Linear & Nonlinear Programming*, McGraw-Hill, 1996.

Nemhauser, Rinnooy Kan, & Todd, eds, *Optimization*, North-Holland, 1989.

Peressini, Anthony L., Francis E. Sullivan, J. Jerry Uhl, Jr. *The mathematics of nonlinear programming*, Undergraduate texts in mathematics series, Springer-Verlag, 1988.

*Mathematical Optimization*, Computational Science Education Project,

<http://csep1.phy.ornl.gov/mo/mo.html>

**Combinatorial Optimization** Foulds would be appropriate as a textbook. Ahuja is a popular text for network flows.

Ahuja, Ravindra K., Thomas L. Magnanti, James B. Orlin, *Network flows : theory, algorithms, and applications*, Prentice Hall, 1993.

Foulds, L. R., *Combinatorial optimization for undergraduates*, Springer-Verlag, c1984.

Lawler, E.L., ed, *The Traveling salesman problem : a guided tour of combinatorial optimization* Wiley, 1985.

Lawler, Eugene L., *Combinatorial optimization : networks and matroids*, Holt, Rinehart and Winston, 1976.

Nemhauser, George L., Laurence A. Wolsey, *Integer and combinatorial optimization*, Wiley, 1988.

Papadimitriou, Christos H., Kenneth Steiglitz. *Combinatorial optimization : algorithms and complexity*, Prentice Hall, 1982.

Parker, R. Gary., Ronald L. Rardin. *Discrete optimization*, Academic Press, 1988.

**Operations Research** These books describe many of the topics above, as part of a broader course in operations research. Generally, the level is below what the course should be taught at.

Ecker & Kupferschmid, *Mathematical models of operations research*

Hillier, Frederick S., Gerald J. Lieberman *Introduction to operations research*, McGraw-Hill, 1990.

Saaty, Thomas L., *Mathematical methods of operations research*, McGraw-Hill, 1959.

Winston, Wayne L., *Operations research : applications and algorithms*, Duxbury Press, 1994.

#### IV. Requirements

Written exercises, written examinations, possibly projects.