MATHEMATICS 280
MULTIVARIATE CALCULUS

(Course number changed from Math 221 Spring 2006)

I. Introduction

A. Catalog Description

This course is a continuation of the calculus sequence that starts with Math 180 and Math 181. It is an introduction to the study of functions with inputs and/or outputs of several variables. The central ideas involving these functions are explored from the symbolic, graphic, numeric and physical model points of view. The themes of visualization, approximation and local linearity from one variable calculus continue to be paramount. Topics include vectors and the basic analytic geometry of three-space; the differential calculus of scalar-input, vector-output functions; the geometry of curves and surfaces; and the differential and integral calculus of vector-input, scalar-output functions. Students will use computer software and graphing calculators to help analyze computationally intensive problems. *Prerequisite: Math 181 or its equivalent*. Satisfies the Mathematical Reasoning core requirement.

B. Objectives

The primary objective for students in this course is to appreciate the power and beauty of the calculus. In this multi-variable setting, students will begin to appreciate the central role of linearity. They will also see how approximation by polynomial functions is fundamental for understanding optimization problems. In the study of vector-input, vector-output functions, students will see how abstract concepts lead to a deeper understanding of important mathematical ideas. In particular, they will begin to experience the organic and highly interconnected nature of mathematics by using calculus to analyze and solve problems from the sciences and business related fields.

This course satisfies the Mathematical Approaches category of the university's core curriculum by developing an appreciation of the power of Mathematics and formal methods to provide a way of understanding a problem unambiguously, describing its relation to other problems, and specifying clearly an approach to its solution. A student in this course will develop a variety of mathematical skills, an understanding of formal reasoning, and a facility with applications. Specifically, this course will develop the study of formal logic, at least to the extent that is required to understand mathematical proof.

C. Prerequisites - Math 181 or its equivalent.

II. Required Topics

A. Vectors and analytic geometry of three-space

1. Component and geometric views of vectors
2. Vector algebra
3. Dot and cross products
4. Planes and lines in $\mathbf{R}^3$

II. Required Topics (cont.)

B. Differential calculus of scalar-input, vector-output functions
   1. Visualizing scalar-input, vector-output functions
   2. Limit, continuity and derivative
   3. Arc length
   4. Geometry of plane and space curves

C. Differential calculus of vector-input, scalar-output functions
   1. Visualizing vector-input, scalar-output functions: slices and level curves
   2. Limits and continuity
   3. Partial derivatives, gradient, and directional derivatives
   4. Higher order partial derivatives and Taylor polynomial approximations
   5. Extrema problems: unconstrained and constrained

D. Integral calculus of vector-input, scalar-output functions
   1. Double integrals and iterated integrals
   2. Double integrals in polar coordinates
   3. Triple integrals and iterated integrals
   4. Triple integrals in cylindrical and spherical coordinates

E. Differential calculus of vector-input, scalar-output functions
   1. Visualizing vector-input, scalar-output functions: slices and level curves
   2. Limits and continuity
   3. Partial derivatives, gradient, and directional derivatives
   4. Higher order partial derivatives and Taylor polynomial approximations
   5. Extrema problems: unconstrained and constrained

F. Calculus of vector-input, vector-output functions
   1. Curl and divergence
   2. Line integrals
   3. Surface integrals
   4. Fundamental Theorems of Calculus: Stokes', Green's, Gauss'.

III. Bibliography

1. G. L. Bradley & K. J. Smith, Calculus
2. Thomas Dick & Charles Patton, Calculus of Several Variables
3. W. G. McCallum, D. Hughes-Hallett et al., Multivariable Calculus
4. J. Marsden, A. Tromba, & A Weinstein, Basic Multivariable Calculus