Math 300

Final Exam

Name _____

I. Logic and foundations

    a.   (10 pts.)   Simplify the following by moving negations in as far as possible

        1. $\neg \forall x (\Phi(x)$

        2. $\neg (p \vee \neg q)$

    b.   (10 pts.)   What is a proof?

c.   (10 pts.)   What is an axiom? Give two distinct points of view on
      this.

d.   (10 pts.)   What are the components of an axiomatic system?
Illustrate your answer by giving examples of these components drawn from our
study of geometry.

e.   (5 pts.) What does it mean for an axiom system to be categorical? Is
      Euclidean geometry categorical?

f.   (15 pts.)   What is the Metamathematical Theorem and how do we go
      about proving it?

g.   (10 pts.)   Give brief definitions of **intuitionism** and **logicism**.

h.   (10 pts.)   How do we prove something by using a *reductio ad absurdum* proof?

i.   (10 pts.)   What does it mean to say that an axiom is independent of a given set of axioms? Give an example we have looked at this term.

j.   (10 pts.)   Briefly describe the Poincaré disk model of hyperbolic geometry, giving interpretations of the undefined terms.

k   (10 pts.)   Suppose that we have two points in the Poincaré disk model A and B, and that the P-line through them extends from points C and D on the bounding circle of the Poincaré model. What is the formula for calculating the Poincaré distance between A and B?

II.   Geometry

a.   (15 pts.) Using a careful statement/justification format, prove that for every point there exists a line not incident with that point. (Prop 2.4)

b.   (15 pts.) Do one (but only one) of the following:

1.   Prove that given a triangle ABC there is at least one vertex from which the perpendicular dropped from that vertex to the opposite side meets the opposite side in a point between the two endpoints of the opposite side.

2.   (problem 15, page 138). Given a triangle ABC. Let D be the midpoint of BC and E the unique point on the line through A and D such that A*D*E and AD congruent to DE. Draw a diagram and show that the angle sum of triangle ABC is the same as triangle AEC.

c.   (15 pts.) Do one (but only one) of the following:

1.   Prove that the sum of the degree measures of any two angles in a triangle
          is less than or equal to the degree measure of their remote exterior angle
          (corr. 1 to the Saccheri-Legendre theorem)
2.   Prove that if triangles are without defect, then a triangle inscribed in a
          semi-circle is a right triangle.
3.   Prove the Universal Hyperbolic Theorem.

III.   Some History

1.   (20 pts.)   Write brief paragraphs on two of the following persons saying
        what their contribution to geometry was:

Farkas Bolyai	Gauss	Lobachevsky
János Bolyai	Hilbert	Poincaré
Clairaut	Klein	Proculus
Dedekind	Lambert	Saccheri
Euclid	Laplace	Thales
Frege	Legendre	

IV   Final questions and constructions.

1.   (10 pts.)   What, in your opinion, is the most beautiful theorem in this
        course? Why?

1.   (15 pts.)   In the Poincaré disk model for hyperbolic geometry, construct the
     P-line through the two points indicated below by cross-hairs. The center is also
     indicated by cross-hairs.