Math 300

Second Hour Exam

Name ________________________________

Friday, March 23, 2007
(I have included one of the propositions in the list of axioms)

1. Some definitions (5 points each). Give formal definitions of the following:

a. Neutral geometry. Why do we study this?

b. Line separation property

c. Congruence of triangles

d. Remote exterior angle to an interior angle of a triangle (drawing and labeling a picture is sufficient)

e. Dedekind Cut

2. (10 pts.) How do we know that parallel lines exist in neutral geometry? What does this say about elliptic geometry? How does this explain the way Hilbert stated his parallel axiom?

3. (10 pts.) In talking about elementary continuity we talked about points inside and points outside a circle. Give a definition to say that a point P is inside a circle with center O and radius OR, and a second one to say that a point P is outside a circle with center O and radius OR.

4. (15 pts.) Suppose that A*B*C and B*C*D. Show that A*B*D (i.e., prove part of the corollary to Proposition 3.3). Hint: Let E be a point not on the line through B and C (how do we know one exists) and consider the line through E and C.

5. (5 pts.) State the Exterior Angle Theorem (Theorem 4.2)

6. (10 pts.) Show that if A*B*C then the segment AB is a subset of the segment AC.

7. (10 pts.) Consider $\Sigma_1 = \{x \in R \mid (x < 0) \vee (x^2 < 2)\}$ and $\Sigma_2 = \{x \in R \mid x^2 >= 2\}$. Show that this is a Dedekind cut. Why would we be interested in this?

8. (5 pts.) Who was David Hilbert?

9. (10 pts.) One of the exercises we did with straight-edge and compass was to make a copy of a triangle.(Major exercise 1.f from chapter 1). We proved that we could do this as a corollary to CA 6 (SAS). Why was it important that we be able to prove that this construction could be done using Hilbert's axioms?

Axioms and major propositions
(These pages may be removed and need not be returned with the exam. Please make sure that you don't tear off any exam questions!)

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

EP5 (Euclidean Parallel Postulate): If two lines are cut by a common transversal making interior angles on one side of the transversal whose degree measures add up to less than 180 degrees, then the two lines meet on that side of the transversal..

LR1: The following are the six types of justifications allowed for statements in proofs:
(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at least two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A

BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.

BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.

BA4: For every line l and for any three points A, B, and C not lying on l,

(i) if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l

(ii) If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l

(corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l

CA1: Copying segments onto rays

CA2: Congruency of segments is an equivalence relation

CA3: Addition of segments

CA4: Copying angles

CA5: Congruency of angles is an equivalence relation

CA6: SAS

Circular Continuity Principle: if a circle C has one point inside and one point outside another circle C', then the two circles intersect in two points.

Elementary Continuity Principle: If one end point of a segment is inside a circle and the other outside, then the segment intersects the circle.

Archimedes' Axiom: IF CD is any segment, A any point, and r any ray with vertex A, then for every point B != A on r there is a number n such that when CD is laid off n times on r starting at A, a point E is reached such that nCD is congruent to AT and either B = E or A*B*E.

Aristotle's Axiom: Given any side of an acute angle and any segment AB there exists a point Y on the given side of the angle such that if X is at the foot of the perpendicular from Y to the other side of the angle, XY > AB.

Dedekind's Axiom: Suppose that the set *{l}* of all points on a line *l* is the disjoint union $\Sigma_1 \bigcup \Sigma_2$ such that no point of either subset is between two points of the other. Then there exists a unique point O on *l* such that one of the subsets is equal to a ray of l with vertex O and the other subset is equal to the complement.

Selected propositions / theorems

Proposition 3.3. Given A*B*C and A*C*D. Then B*C*D and A*B*D

Corr: Given A*B*C and B*C*D. Then A*B*D and A*C*D.