Math 300

First Hour Exam

Name ______________________________

Friday, Feb. 16

I. Some definitions (5 points each). Give formal definitions of the following.

a. A segment

b. A right angle

c. A model

d. A projective plane.

II. (10 pts.) Below draw a (straight) line and select, some point not on the line. Then using straight-edge and compass only, draw a line through the given point perpendicular to the line. Show all marks (if I can't see how you did it, you may not get credit for it).

III. (10 pts.) Fill in the gaps in the following proof. Logic rules and the axioms of incidence geometry are given on the last page of this exam.

Given: A point P
To show: That there is a line not passing through it.

Number	Statement	Justification
1	Let P be a point	
2.	Suppose that all lines pass through P	
3.	There exist three distinct lines that are not concurrent; call them l, m, and n	Prop. 2.2
4.	l, m, and n do not all pass through any given point	
5.	l, m, and n all pass through P	
6.	Therefore the assumption in 2 is false, and there is a line not passing through P.	

IV. (10 pts.) prove that given distinct points A and B, $AB \subseteq \overrightarrow{AB} \cap \overrightarrow{BA}$

V. (5 pts. each). Given the statement "If the sun breaks through it gets warm". Give, in ordinary English:

a. The converse to the statement

b. The contrapositive of the statement

c. The negation of the statement

d. What is the sufficient part?

e. What is the necessary part?

VI. (5 pts. each) Writing the negation of a statement is a useful thing to be able to do. In part (a), write down the negation of the statement, simplifying it so that negations occur only before predicates. In part (b), state the negation in conversational English.

a. $\exists x(p(x) \wedge \neg q(x))$

b. All right angles are congruent

VII (10 pts.) One of the rules of logic asserts that not (p and q) is equivalent to (not p) or (not q). Use truth-tables to demonstrate this (the following diagram should get you started). What in the completed truth table tells you that you should believe the statement?

p	q	p and q	not (p and q)	not q	not p	(not p) or (not q)

VIII (5 pts.) Say something (a brief sentence - something relevant to the course) about three of the following names.

a. Euclid
b. Thales of Miletus
c. David Hilbert
d. Proclus Diadochus
e. Gottlob Frege
f. Adrien Marie Legendre
g. Pythagoras of Samos

LR0: No unstated assumptions.

LR1: The following are the six types of justifications allowed for statements in proofs:
(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at lease two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.
: For every point P there exist at least two lines through P

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A
BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.
BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.