# Honors 213


# Final Exam


**Name _____**

Monday, May 8, 2000
200 points

1.      (20 pts.)        In this course we have given some thought to axiomatic systems: systems of undefined terms, axioms, and rules of reasoning.  In the context of this discussion, describe what an axiom is, and say how the meaning of the term has changed from Euclid to Hilbert.

2.     (20 pts.).     In this course we have also had some experience with proofs.  Give a brief definition of a proof, and list (and briefly describe) some properties that an argument must have in order to be called a proof.

3.    (20 pts.).    Scorpling Flugs.  The following is taken from Trudeau's book
      (Trudeau, Richard J.:  The Non-Euclidean Revolution.  Birkhäuser, 1987)

Undefined terms:  scorple, flug
Axioms:
      SF1:  If A and B are distinct flugs, then A scorples B or B scorples A (or both)
      SF2:  No flug scorples itself
      SF3:  If A, B, and C are flugs such that A scorples B and B scorples C
            then A scorples C.
      SF4:  There are exactly four flugs.

Prove that:  If a flug scorples another, it is not also scorpled by the other (i.e., the "or
      both" of SF1 can not happen).

Draw a picture of a model involving four blocks on a table top.  What interpretations
      have you given the undefined terms?

4.      (20 pts.)        The following is adapted from Lewis Carroll:  Given that

1. No interesting poem are unpopular among people of real taste
2. No modern poetry is free from affectation
3. All Dale's poems are on the subject of soap-bubbles
4. No affected poetry is popular among people of real taste
5. No ancient poetry is on the subject of soap-bubbles

Give a proof that all of Dale's poems are uninteresting.  Hint:  begin by translating the statements into more typical if-then statements (for example, 4 might become "If X's poetry is popular among people of real taste, then it is not affected poetry").  You may need to add some "definition steps", for example that if a poem is not ancient then it is modern.  Justify your steps.

5.      (20 pts.).      Give brief definitions of **interpretation** and **model**, distinguishing between the two.  Use the Poincaré disk model for hyperbolic geometry to illustrate.

6.      (10 pts.)      What is neutral geometry?  Is Euclidean space (that is, a space that is a model of Euclidean geometry) a model of neutral geometry?  Is hyperbolic space a model of neutral geometry?

7.      (10 pts.).      What do we know about the existence of parallel lines

a.      In neutral geometry?

b.      In  Euclidean geometry?

8.      (15 pts.)        What do we know about the existence and variety of parallel lines in hyperbolic geometry?  Illustrate your answer using the Poincaré disk model.

9.      (15 pts.)        Give a definition of an inverse of a point with respect to a circle, and illustrate (by drawing a diagram) how one can construct the inverse of a point inside a circle (and not the center of the circle).

10. (15 pts.)   Given a circle A, a point P, and the inverse P' of P (with respect to the circle A).  Let B be a circle that passes through P and P'.  What is true about the intersection of A and B?  How does this demonstrate that Incidence Axiom 1 is satisfied in the Poincaré disk model?

11. (10 pts.)   Define the Poincaré length of a Poincaré segment AB in the Poincaré disk model (draw a diagram to illustrate your definition).  Use this to give an interpretation of congruence of segments in the Poincaré disk model.

12. (10 pts.) Given that A, B, and C are points in the Poincaré disk model with A*B*C, show that if d(AB) is the Poincaré length of the segment AB, then d(AC)=d(AB)+d(BC). What axiom does this result help us to verify? Hint: recall that log(xy) = log(x)+log(y).

13. (15 pts.) Given a statement of and an outline of the proof of the "metamathematical theorem". Additional space is available on the next page if necessary.

Workspace for problem #13.