Honors 213

First Hour Exam

Name _________________________________

Friday, Feb. 4, 2000
100 points

I. Some definitions (5 points each). Give formal definitions of the following:

a. A ray

b. An angle

c. A right angle

d. A midpoint of a segment

II. Give more informal descriptions of the following (5 pts. each)

a. A postulate

b. Having the elliptic property (said of a model)

III. (5 pts. each). Given the statement "If I go to the coffee shop then I drink a mocha". Give, in ordinary English:

a. The converse to the statement

b. The contrapositive of the statement

c. The negation of the statement

IV. (5 pts. each) In the statement in part III,

a. What is the sufficient part?

b. What is the necessary part?

V. (10 pts.) Give, in English, the negation of incidence axiom 2 as you would use the negation in a proof (carry this through: it is not sufficient to say "It is false to assert that ...."

VI (15 pts.) One of the rules of DeMorgen asserts that not(p and q) is equivalent to (not p) or (not q): that is,

$$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$$

Use truth-tables to demonstrate this (the following diagram should get you started). What in the completed truth table tells you that you should believe the statement?

p	q	not p	not q	p and q	not (p and q)	(not p) or (not q)

VII Models and why they are useful

a. (10 pts.) Give a definition of an interpretation and a model.

b. (10 pts.) How could a model demonstrate that the Euclidean parallel postulate did not follow from the other postulates of Euclidean geometry? Explain how this would work.

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

Euclidean Parallel Postulate: For every line **l** and for every point P that does not lie on **l** there exists a unique line m through P that is parallel to **l**.

LR1: The following are the six types of justifications allowed for statements in proofs:

(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).