Math 280 B

FOURTH HOUR EXAM

NAME____

General Notes:

1. **Show work**.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of a number) and leave it in that form.

Friday, Dec. 4, 2009
95 pts. (will be normalized to 100
pts. in the gradebook.

I. Integration

1. Consider the function $(x + y)^2 = x^2 + 2xy + y^2$ over the region bounded by the positive x and y axis and the line y = 1 - x.

   a. (5 pts.) Sketch the region.

   b (5 pts.) Give two separate ways (integration limits) to describe the region

(problem 1 continued)

c. (10 pts.) Pick one of those two ways and evaluate the integral for the given region: That is, evaluate

$$\iint_A (x+y)^2 \, dA \text{ for this region}$$

d. (10 pts.) Now suppose that the function $(x+y)^2 = x^2 + 2xy + y^2$ is a density function. Calculate the x coordinate of the center of mass for the region. Please note that you have done part of the problem in part (c) above.

II. Gradient fields.

a. (10 pts.) Find the gradient field of the vector function $f(x, y, z) = e^{xy}\sin(z) + xyz$

III. Line integrals and work.

a. (5 pts.) Give a definition of the work done by a force $\vec{F} = M(x, y, z)\hat{i} + N(x, y, z)\hat{j} + P(x, y, z)\hat{k}$ in moving an object over a smooth curve $\vec{r}(t)$ from t=a to t=b

b. (5 pts.) Now give a useful form (using a differential form, as it happens) of the definition (one which you would rather use for the following problem)

(problem III continued)

c. (15 pts.) Calculate the work done in moving an object against the force field given by $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ along the path given by $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + 3t\hat{k}$ by calculating the appropriate line integral. As you know, there is a simpler way to do this without calculating a line integral, but use that simpler way only for checking your work.

d. (5 pts.) When the force field F in the preceding problems is replaced by a continuous velocity field, what do we call the corresponding line integral?

IV. Flux

a. (5 pts.) Suppose that **C** is a smooth closed curve in the domain of a continuous vector field $\vec{F} = M(x, y)\hat{i} + N(x, y)\hat{j}$. What is the definition of the **flux** of F across **C**?

b. (5 pts.) What is a convenient calculation formula for flux (using a differential form again).

c. (15 pts.) Calculate the flux of the vector function $\vec{F}(x, y) = x\hat{i} - y\hat{j}$ across the circle of radius 2 centered at the origin oriented in a counter-clockwise direction.