Math 210

Fall 2008

Final Exam

Name ______________________________

Wednesday, Dec. 17, Noon - 2:00
200 pts.

I. Logic and proof

1. (10 pts.) Given that G = (V, E) is an undirected graph, consider the statement
"**If** G has **e** edges **then** $2e = \sum_{v \in V} \deg(v)$ "

What is the sufficient condition?

What is the necessary condition?

What is the converse of the statement?

What is the contrapositive of the statement?

2. (10 pts). Construct a truth table for $(\neg(p \wedge q) \wedge q) \rightarrow \neg p$.

II. Some set theory

1.. (10 pts.) Prove that for any sets A, B, and C that $A \bigcap (B \cup C) = (A \cap B) \cup (A \cap C)$ by showing that each side of the equality is a subset of the other.

III. Algorithms

1. (10 pts.) Give a definition of an algorithm.

2. (10 pts.) Give a brief definition of "big-O. First give a mathematical definition, followed by an informal (conversational English language) definition.

3. (5 pts.) A propositional statement is said to be **satisfiable** if we can find an assignment of truth values to the symbols in the expression (such as p, q in problem 2 on page 3) which makes the entire expression true. One way to do this is to build a truth table for the expression. If we have n distinct symbols in the expression (as we have two in problem 2 on page 3), how many rows will the resulting complete truth table have? From this, give a "Big-O" estimate of the satisfiability problem (as a function of the number of the number of distinct symbols in the expression).

4. (5 pts.) Give a justification for the statement that the bubble sort has $O(n^2)$ data comparisons and data moves.

IV. Induction

1. (15 pts.) Using mathematical induction, prove that $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2} \right)^2$. **Carefully describe each step in the proof.**

V. Counting and such

1. (10pts.) What is the coefficient of x^5 in the expansion of $(2x-y)^{10}$?

2. (5 pts.) What is a combinatorial proof?

3. (10 pts) Give a combinatorial proof that $\binom{n}{r} = \binom{n}{n-r}$

VI. Some basic number theory

1. (5 pts.) Define a|b where a and b are integers.

2. (5 pts.) Define $a \equiv b \bmod n$ (a is congruent to b mod n, where a, b, and n are integers).

3. (10 pts.) Fill in the following multiplication table for integers mod 3. In each square place the smallest positive integer equal to the product.

*	0	1	2
0			
1			
2			

4. (15 pts) What is the internal (hexadecimal) representation of the integer -25? Please note that this problem asks you to (1) construct the binary representation of 25, (2) take the 2's complement, and (3) convert that two's complement into hexadecimal.

VII. Some basic probability

1. (5 pts.) Define P(A|B)

2. (5 pts.) State Bayes' theorem

3. (10 pts.) A Bernoulli trial with probability of success = 0.75 is performed 10 times. What is the probability of exactly 4 successes?

4. (5 pts.) What is the probability that a five-card poker hand will contain the King and Queen of hearts?

VIII Graphs

1. Some definitions (10 pts.)

Simple graph

Degree of a vertex (in a simple graph)

Adjacency of two nodes in a graph

What is the handshaking theorem?

2. (10 pts.) The following matrix is the adjacency matrix of a graph. Draw the graph represented by the matrix:

$$A=\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. (10 pts.) Squaring the matrix of the previous problem (as a regular matrix, not as a zero-one matrix) counts the number of paths of length 2 from one node to another. What is the square A*A of the matrix of the preceding problem?

IX. History (10 pts.) Pick four of the following names of people and say something about them. Clearly indicate which persons you are describing (duplicates do not count if I have accidentally included one).

a) René Descartes
b) Georg Cantor
c) Leonardo of Pisa
d) Karl Friedrich Gauss
e) Charles Dodgson
f) Paul Gustav Heinrich Bachman
g) Ada Augusta, Countess of Lovelace
h) Marin Mersenne
i) James Bernoulli
j) Pierre-Simon Laplace
k) Abu Ja'far Mohammed Ibn Musa Al-Khowarizmi
l) Donald Knuth
m) George Boole
n) Pierre de Fermat
o) G. Lejeune Dirichlet
p) Pierre-Simon Laplace
q) Paul Erdös