Math 210

Third Hour Exam

Name ______________________________

No calculators should be necessary for this exam

Friday Nov. 14
100 points

I. Some matrix problems (5 pts. each)

Consider the two matrices $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

1. Calculate the sum of the two matrices

2. Calculate the matrix product of the two matrices

Now notice that the two matrices on the previous page are zero - one matrices, and work the following two problems (recall that the two matrices are

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

3. Calculate the meet ($\wedge$) of the two matrices

4. Calculate the Boolean product of the two matrices.

II. Induction and recursion. Recall that the Fibonacci sequence is defined by

$f_0 = 0$

$f_1 = 1$

$f_n = f_{n-1} + f_{n-2} \forall n > 1$

Suppose we want to show that $f_1^2 + f_2^2 + ... + f_n^2 = f_n f_{n+1}$ using mathematical induction. (this is problem 12 on page 308 of the textbook for future reference)

1. What is the base case? (5 pts.)

2. Prove the base case (5 pts.)

(continued on next page)

(continuation of problem II)

3. What is the inductive step? (5 pts.)

4. What is the inductive hypothesis? (5 pts.)

(continuation of problem II)

5. Prove the inductive step. (10 pts.)

6. (10 pts.) In your favorite language, write a recursive routine to calculate the nth Fibonacci number.

III. Some counting questions

1. (5 pts.) A **nybble** is a sequence of 4 bits. How many are there? What principle of counting are we using?

2. (5 pts.) A student can complete a unit by either doing one of three projects or one of two exams. In how many ways can the student complete the project? What principle of counting are we using?

3. (10 pts.) A part number consists either of a two uppercase letters followed by three digits or by two digits followed by three upper case letters.. How many part numbers can we construct? Recall that there are 26 upper case letters.

4. (5 pts) State a version of the pigeon hole principle.

5.. (10 pts.) Show that if we throw a dice (values 1 - 6) seven times, at least one of the numbers will show up twice.

IV. (5 pts.) Say something appropriate about one of the following:

a) James Bernoulli
b) Gabriel Lamé
c) Fibonacci (Leonardo of Pisa)
d) G. Lejeune Dirichlet
e) Marin Mersenne