Math 210

First Hour Exam

Name _____

Friday, September 28
100 pts.

1.	(10 pts.) Let p be the statement "it is sunny" and q the statement "we go on a picnic". Translate the following into symbolic form:

a)	If it is sunny we go on a picnic.

b)	It was not sunny, but we went on a picnic..

c)	We go on a picnic only if it is sunny..

2.   (10 pts.) Given the statement "If it is sunny we go on a picnic" What is the

Sufficient condition?

Necessary condition?

Converse?

Contrapositive?

3.   (10 pts.) Simplify the following expressions by moving the negation sign
        inside so that it appears only directly before predicate expression(s) P(x) and
        Q(x) and so that only **and**, **or**, and **not** are used in addition to the predicates
        and quantifiers (that is, translate implication statements using these three
        symbols). Also remove any double negations.

$\neg \forall x (\neg P(x) \lor Q(x))$

$\neg (\exists x \forall y P(x, y))$

4.   (10 pts.)   Suppose that G(x,y) means that x > y (the domain is the set of integers). Express the following in English (translation, not transliteration: i.e., do not use words like "for all" and "there exists" and "it is not the case that".

a.   What in English does the statement

$\exists x \forall y G(x, y)$

mean?

b.   Negate the statement in part (a) of this problem, and write down what it means in English.

5. (10 pts.) Complete the following truth table for the expression
    $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$. Is the expression a tautology? If so, what rule of logic
    does it verify (for propositional logic)?

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$((p \rightarrow q) \wedge \neg q)$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

6.   (5 pts. each) Let B(x) be the predicate "x is a block in the file", M(x,y) be the
     statement "x wants to modify y", P(x) be "x is a process", A(x) the statement "x is
     available",and L(x,y) the statement "x locks y"

     a.   **Translate** into predicate form the statement "For any x and any file
     block y, if x wants to modify block y and y is available then x locks y."

     b.   Translate into English the statement

             $\forall x \forall y (P(x) \wedge B(y) \wedge L(x, y) \rightarrow \neg A(y))$

7.   (10 pts.) Fill in the justifications (third column) of a proof of the following, giving reasons for each step.

$\forall x(A(x) \rightarrow B(x))$

$\exists xA(x)$

$\therefore \exists xB(x)$

statement 1	$\forall x(A(x) \rightarrow B(x))$	given
statement 2	$\exists xA(x)$	given
statement 3	A(c)	
statement 4	$A(c) \rightarrow B(c)$	
statement 5	B(c)	
statement 6	$\exists xB(x)$	

8.   (15 pts.) Suppose that set A = {a,b}, and that B = {b,c,d}. What is the result of

$A \cup B$

$A \cap B$

$A - B$

$A \times B$

9.   (10 pts.) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

11   (5 pts.) Say something relevant to the course about one of the following:

      a. Henry Maurice Sheffer
      b. George Boole
      c. René Descartes
      d. Augustus De Morgan
      e. Ada Augusta, Countess of Lovelace