

Math 211

Fourth Hour Exam

Name _____

No calculators should be necessary for this exam

Friday Dec. 2
100 pts.

- I. (10 pts.) Consider the following procedure for printing out the moves necessary to move n disks from one peg to another:

```
procedure MoveTowers(IN n, from, to, free: integer) IS
  if (n == 1) then println("move a disk from peg ", from,
    " to peg ", to);
  else {
    MoveTowers(n-1, from, free, to);
    println("move a disk from peg ", from,
      " to peg ", to);
    MoveTowers(n-1, free, to, from);
  }
END MoveTowers;
```

Write a **recurrence** relation to say how many times the **println** statement is executed for n disks in terms of the number of times the **println** statement is executed for $(n-1)$ disks.

2. (15 pts.) Identify each of the following relations as reflexive, symmetric, anti-symmetric, transitive, an equivalence (all that apply):

a. $A \subseteq B$ (for subsets of some set X)

b. xRy (i.e., $(x, y) \in R$) if x and y have taken the same class

c. Graph G is isomorphic to graph H

III. Graph theory (all graphs are to be simple graphs)

1. Some definitions (5 pts. each)

a. A graph

b. Degree of a vertex

c. Path in a graph; a circuit

d. Euler circuit

e. Hamilton circuit

2. (10 pts.) Is it possible have a graph with an odd number of vertices each of degree 3? Why or why not?

3. (10 pts.) What is a graph isomorphism?

4. (10 pts.) From the following adjacency matrix, reconstruct the graph:

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |