

Math 211

Third Hour Exam

Name _____

No calculators should be necessary for this exam

Friday Nov. 11

95 points (will be rounded to 100 in the gradebook)

1. (15 pts.) Pascal's identity holds that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. Write a recursive procedure to calculate $C(n+1, k)$. What are the base cases?

2. Some counting questions (5 pts. each)

a. How many 8 bit sequences of binary digits (bits) can we construct? What principle of counting are we using?

b. A class of 7 students meets in a classroom with 10 chairs. Given one student per chair, how many seating arrangements are there for this class?

c. In how many ways can we pick 6 donuts if we are presented with 4 available kinds of donuts (with at least 6 donuts per kind).

d. We place 125 marbles in 10 bins. One bin will contain at least what number of marbles? What principle are we using?

e. To get to the Institute Henri Poincaré, one travels by airplane to Paris, then by train or bus to Nancy, then either by foot, cab, or bus to the Institute. Suppose that there are three options for airline flight to Paris. How many different ways can we get to the Institute? What principle of counting are we using?

f. What is the coefficient of x^7y^3 in the expansion of $(x + y)^{10}$?

g. How many integers are there between 1 and 10 which are either even or a prime? What principle of counting are we using?

3. Some probability (5 pts. each)

a. Throwing two fair coins, what is the probability of getting one head and one tail?

b. Define $P(A|B)$

c. State Bayes' theorem

d. Suppose that we know the following:

The probability of passing an exam is 70%

60% of students study for exams

The probability of passing an exam if you study is 90%

What is the likelihood that a student studied, given that the student passed the exam?

e. What is the probability of drawing a five-card hand containing exactly three kings?

f. What is a Bernoulli trial?

g. Tossing three fair coins, what is the expected number of heads?

h. Tossing four fair coins, what is the probability that we will get exactly three heads?

3. (5 pts.) Say something appropriate about one of the following:

- a) James Bernoulli
- b) Pierre-Simon Laplace
- c) Fibonacci (Leonardo of Pisa)
- d) G. Lejeune Dirichlet
- e) Blaise Pascal (say something more than to say that he was responsible for Pascal's identity)