Math 180 E

FOURTH HOUR EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, November 30, 2012
100 pts

1. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 15$

(25 pts.) Find the critical points and note over which interval(s) the graph is increasing and decreasing, where it is concave up and concave down and find the points of inflection. Use the second derivative test and concavity (show work) to characterize the critical points as local maxima, local minima, or points of inflection.

a. Critical Points

b. Intervals over which f is

Increasing

Decreasing

(continued on next page)

(continuation of problem 1: $f(x) = 2x^3 - 3x^2 - 12x + 15$)

Calculate the second derivative and use it to determine

c. Intervals over which f is

Concave Up

Concave Down

d. What are the points of inflection (if any) in this case?

e. Use the second derivative to examine the points of inflection from the previous page and, saying why, categorize them (listing them below) as

Local Maxima

Local Minima

2. (10 pts.) Use **l'Hôpital's rule** to find the following limits. **Show your work**:

$$\lim_{x\to\infty} \frac{x^2}{e^x}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos(x)}{(x-\frac{\pi}{2})}$$

3. (5 pts each) Find the following antiderivatives. Remember the constant of integration!

$$\int (4x^2 - 7x + 2)dx$$

$$\int e^{3x}dx$$

$$\int \sin(2x)dx$$

$$\int \frac{dx}{1+x^2}$$

4. (10 pts.) A farmer has 100 yards of fencing and wishes to fence in a rectangular plot up against a straight cliff face. The farmer does not need to fence the cliff face, of course, and needs only to fence in three sides of the rectangular plot. What are the dimensions of the largest rectangular plot that can be enclosed in this manner?

5. (10 pts.) What are the dimensions of the rectangle of maximum area that can be fit into a semi-circle of radius 4? Hint: Draw a picture. If x is the distance from the center of the circle to the right-most side of the rectangle, then the width of the rectangle is $2x$. Use the Pythagorean theorem to write the height of the rectangle in terms of x and the radius (4).

6. (15 points). Calculate R_4 for the function $f(x) = x^2$ on the interval [0, 1]. Carry your calculations through to the point that you could finish the calculation by entering numbers into a simple calculator.

7. (10 pts.) Using the summation rules we have learned so far, calculate

$$\sum_{k=1}^{100} (3x - 2)$$