Math 180

FOURTH   HOUR   EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, April 30, 2010
100 pts

I. Optimization

1. (15 pts.) (Problem #4 on page 276 of the textbook). A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions? Begin by drawing a picture of the situation.

2. (15 pts.)   (From Strauss, Bradley, and Smith <u>Calculus</u>) Suppose that it costs us $C(x) = \frac{1}{8}x^2 + 4x + 200$ dollars to manufacture and distribute **x** units of some commodity, and that we can sell each one for a price of (49-x) dollars per unit for a total revenue R(x) $= x(49 - x)$ dollars for x units. Our profit is then P(x) = R(x) - C(x). For what value of x will we obtain the largest profit?

II. **l'Hôpital's rule**

1. (5 pts. each) Use **l'Hôpital's rule** to find the following limits. <u>Show your work</u>:

$$\lim_{x \to 1} \frac{x^3 - 1}{4x^3 - x - 3}$$

$$\lim_{x \to \infty} x^2 e^{-x}$$

III. Newton's Method and the Method of Bisection

1. (15 pts.)   Suppose that we want to solve the equation $x^3 - x = 0$.

    a. There is a solution between 1 and 2. If we use the method of bisection, we set low to 1 and high to 2. What is the next value of low and high? That is, starting with the fact that a solution lies in the interval [1, 2], what is the next interval we try?

    b. We use Newton's method with an initial guess of 2. What is the next guess?

IV. Antiderivatives (5 pts each) Find the following antiderivatives. Remember the constant of integration!

$$\int (2x^5 + 4x^3 - 7x^2 + 1)dx$$

$$\int \sin(x)dx$$

$$\int e^x dx$$

$$\int \sec(x)\tan(x)dx$$

$$\int \frac{dx}{1+x^2}$$

V. Summations

1. (10 pts.) Evaluate $\sum_{k=1}^{100} (6k^2 + 4k + 3)$ to a number using the rules and formulae we have developed. Show your work.

VI. The definite integral

    1. (10 pts.)   What is a partition of a closed interval [a, b] (definition)? What is the norm of a partition?