Math 180

THIRD HOUR EXAM

NAME____

General Notes:

1. **Show work**.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of the logarithm of a number) and leave it in that form.

Tuesday, November 16, 2010
90 pts. (will be adjusted to 100 points in the gradebook)

I. Inverses (5 pts each)

a. Find $ArcCos(\frac{\sqrt{3}}{2})$ (inverse cos)

b. $\frac{d}{dx}ArcTan(x) =$

2. Chain Rule (5 pts. each)

$$\frac{d}{dx}(x^2+2x+3)^{15} =$$

$$\frac{d}{dx}e^{\sin(x)} =$$

$$\frac{d}{dx}e^{\ln(x)} =$$

2. Approximations and rates of change

a. (5 pts.) What is the standard linear approximation to the function $f(x) = \sqrt{1+x}$ at the point x = 0? Use it to approximate the square root of 1.01. (problems 16, 17, page 218)

b. (10 pts.) An object is dropped from the top of a 100 meter high tower. Its height above the ground after t seconds is $100 - 4.9t^2$. How **fast** is it following 9 seconds after it is dropped? What is its **acceleration** at the time? What is its **jerk** at that time?

(15 pts.) Sand is being dropped by a conveyor belt onto a conical pile which is always twice as high as the base (a circle) is wide at the rate of 10 cubic feet / minute. How fast is the height of the pile increasing when the height is 18 feet (and the base has a diameter of 9 feet)? Recall that the volume of a right circular cone is given by $V = \frac{\pi}{3} r^2 h$. Hint: use similar triangles.

3. Mean Value Theorem and some applications.

a. (5 pts.) State the Extreme Value Theorem (with preconditions)

b (5 pts.) State the Mean Value Theorem (with preconditions)

c. (5 pts.) Verify the mean value theorem for the function $y = x^2 + 1$ on the interval [1, 2] (i.e., find a number c in the interval (1, 2) which satisfies the conclusion of the mean value theorem in this case.

4 Implicit differentiation

a. (20 pts.) Find the equation of the lines tangent and normal to the curve $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$ by first finding the slope at that point $\frac{dy}{dx}$ using implicit differentiation and then using the slope and the point to find the tangent and normal lines at that point.