Math 180 C

FINAL EXAM

NAME____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Wednesday, May 12, 2010
200 pts

I. Limits and Continuity

1. (15 pts.) Define (give an ε-δ definition) $\int_a^b f(x)dx$

2. (10 pts.) Now describe what this means as if writing to a friend who has not seen this stuff.

3. (5 pts.) Suppose that we know that f(x) is continuous on [a, b] and that f(a) < 0 and that f(b) > 0. Can we say that f(x) = 0 for some x in [a, b]? Why or why not?

5. (5 pts.) State the extreme value theorem for continuous functions.

6. (10 pts.) Define $f'(x) = \frac{df}{dx}$ as a limit

7. (5 pts.) A car drives on a road. What does the number indicated by speedometer have to do with a derivative?

7. (15 pts.) Find all asymptotes and the x and y intercepts of the function $y = \frac{x^2 - 4}{x - 1}$ and give a brief sketch of the graph of the function.

II. The Derivative as a Function

1. (**5** pts. each) Evaluate the following derivatives

$$(4x^2+7x-4)^7$$

$$x\sin(x)$$

$$\frac{\cos(x)}{e^x}$$

$$e^{\cosh(x)}$$

2. (10 pts.) compute the derivative of the following function using logarithmic differentiation:

$$\left(\frac{1}{x-1}\right)\left(\frac{x}{x^2+1}\right)$$

3. (15 pts.) Find the equation of the lines tangent and normal to the curve defined by $\frac{x^2}{9}+\frac{y^2}{25}=1$ at the point $(\frac{9}{5},4)$

III. Applications of Derivatives (remember to show your work)

1. (10 pts.) A sphere is inflated at the rate of 10 cubic inches a second. When the radius of the sphere is 10 inches, how fast is the surface area increasing? The surface area of a sphere of radius r is given by $S = 4\pi r^2$

2. (15 pts.) (problem 12 on page 277) What are the dimensions of the open-top right circular cylinder with volume 1000 cubic centimeters and the smallest possible surface area (cylinder and bottom). Remember that the area of such a cylinder would be made up of circle of radius r at the base (πr^2) and the walls of the cylinder ($(2\pi rh)$ where r is the radius of the can and h its height.

IV. Integration

1. Evaluate the following definite and indefinite integrals (**7** pts. each, 35 points total) Evaluate definite integrals to a number, and don't forget the constant of integration on indefinite integrals!

a. $\int_0^1 (x^3 - x)dx$

b. $\int_0^{\frac{\pi}{3}} \cos(\theta)d\theta$

b. $\int_0^{\ln(3)} e^{2x}dx$

c. $\int_0^1 \frac{dx}{1+x^2}$

d. $\int xe^{x^2}dx$

2. (10 pts.) What is the average value of $f(x) = x^3$ on the interval [0,2]?

3. (10 pts.) Evaluate (using the rules and formulae we have studied) $\sum_{1}^{10}(2k-1)$

4. (10 pts.) Evaluate

$$\frac{d}{dx}\int_1^x \frac{1}{t}dt$$