Math 180 A

FINAL EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Monday, May 10, 2010
200 pts

I. Limits and Continuity

1. (10 pts.) Define (give an ε-δ definition) $\lim_{x->a} f(x) = L$

2. (10 pts.) Now describe what this means as if writing to a friend who has not seen this stuff. Please note that it is not sufficient to transliterate your definition above or to say "As x goes to a", etc.)

3. Define (5 pts. each)

a. f is continuous at a point x_0

b. f is continuous on the interval [a, b]

4. (5 pts.) State the intermediate value theorem for continuous functions

5. (5 pts.) Verify the mean value theorem for functions for the function $f(x) = x^3$ on the interval [0, 1]

6. (10 pts.) Define f'(x) = $\frac{df}{dx}$ as a limit

7. (5 pts.) What is the geometrical interpretation of a derivative?

7. (15 pts.) Identify all critical points of the function $y = 2x^3 - 3x^2 + 6$ and use the second derivative test to identify them as local maxima or local minima. Use the first derivative to determine where the function is rising and where it is falling.

II. The Derivative as a Function

1. (**5** pts. each) Evaluate the following derivatives

$(3x^2 + 2x + 1)^4$

xe^x

$\frac{e^x}{\cos(x)}$

$\cosh(2x + 1)$

2. (10 pts.) Compute the derivative of the following function using logarithmic differentiation:

$$y = (\frac{1}{x-1})(\frac{x}{x^2+1})$$

3. (10 pts.) Find the equation of the line tangent to the curve defined by $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at the point $(5, \frac{16}{3})$

III. Applications of Derivatives

1. (15 pts.) A conveyor belt loads sand onto a conical pile (a right circular cone) at the rate of 10 cubic feet / minute. The pile always maintains a ratio of height to **diameter** of the base of 1 to 1 (for example, when the pile is 10 feet high the base has a **diameter** of 10 feet as well). How fast is the height of the cone changing when the height is 10 feet?

The volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$

2. (10 pts.) A rectangular field is to be fenced which happily is bordered on one side by a surprisingly straight river. Given 100 feet of fencing (none of which is needed on the riverside), what are the dimensions of the largest rectangle which can be enclosed by the given fencing?

IV. Integration

1. Evaluate the following definite and indefinite integrals (**7** pts. each, 35 points total) Evaluate definite integrals to a number, and don't forget the constant of integration on indefinite integrals!

$$\int_0^1 (x^3 - x)dx$$

$$\int_0^{\frac{\pi}{3}} \cos(\theta)d\theta$$

$$\int_1^{\ln(5)} e^x dx$$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$\int x\cos(x^2)dx$$

2. (10 pts.) What is the average value of $f(x) = x^2$ on the interval [0,2]?

3. (10 pts.) Evaluate (using the rules and formulae we have studied) $\sum_{1}^{10}(2k+1)$

4. (10 pts.) Evaluate

$$\frac{d}{dx}\int_{1}^{x}\frac{1}{t}dt$$