Math 180 F

FINAL   EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Monday, Dec. 14, 2009
200 pts

I. Limits and Continuity

1. (10 pts.) Define (give an ε-δ definition) $\lim_{x \to a} f(x) = L$

2. Define (5 pts. each)

    a. f is continuous at a point x_0

    b. f is continuous on the interval [a, b]

3. (5 pts.) State the intermediate value theorem for continuous functions

4. (5 pts.) Suppose that f is continuous on the closed interval [a, b]. What can we say about the maximum and minimum values of f on [a,b]?

5. (10 pts.) Define $f'(x) = \dfrac{df}{dx}$ as a limit

6. (15 pts.) Find all asymptotes and the x and y intercepts of the function $y = \dfrac{x^2 - 1}{x + 2}$ and give a brief sketch of the graph of the function.

II.   The Derivative as a Function

1.   (**7** pts. each, 35 points total) Evaluate the following derivatives
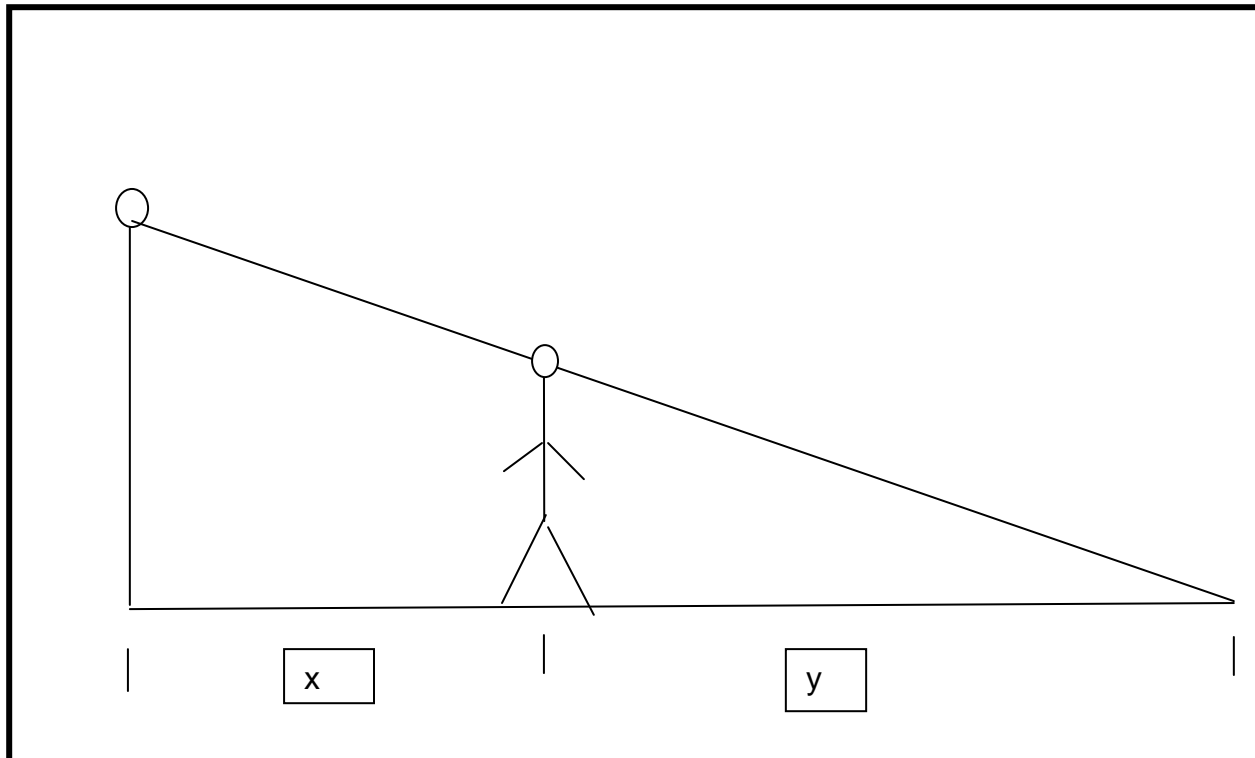
$$(x^3 - 4x^2 + 7x - 4)^{10}$$

$$e^x \sin(x)$$

$$\frac{2x+1}{3x-1}$$

$$e^{\sin(x)}$$

$$x^x$$

2. (15 pts.) A person 6' tall walks away from a streetlamp 10' tall at the rate of 4 ft/sec, casting a shadow. How fast is the length of the shadow lengthening when the person is 6' from the lamp? Note that because of similar triangles, $\dfrac{x+y}{10} = \dfrac{y}{6}$

3.   (10 pts.)   **Using logarithmic differentiation**, find y' for $y = \left(\dfrac{1}{x-1}\right)\left(\dfrac{1}{x-2}\right)\left(\dfrac{1}{x^2+1}\right)$

III.   Applications of Derivatives

1.   In a certain manufacturing business, the revenue earned by the sale of x units (which could be thousands of parts) is given by $r(x) = -x^3 + 7x^2 - 5x + 1$ and the cost of manufacturing x units is given by $c(x) = 5x + 1$.

    a.   (5 pts). What is the marginal revenue as a function of x in this case?

(problem 1 continued)

b.   (15 pts.)   The profit function p(x) = r(x)-c(x). For what value of x will this be a maximum? For what value of x will profit be a minimum? Use first and second derivatives to find these values. You should find two critical points. Note - you will need to use the quadratic formula and leave your answers in radical form.

c.   (10 pts.)   Consider the revenue function $r(x) = -x^3 + 7x^2 - 5x + 1$ again. On which intervals is the function concave up? On which intervals is the function concave down?

IV.   Integration

1.   Evaluate the following definite and indefinite integrals (**7** pts. each, 35 points total)
    Evaluate definite integrals to a number, and don't forget the constant of integration on
    indefinite integrals!

a. $\int_{0}^{1}(x^3 - x)dx$

b. $\int_{0}^{\frac{\pi}{3}}\cos(\theta)d\theta$

b. $\int_{0}^{\ln(3)}e^{x}dx$

c. $\int\frac{dx}{1+x}$

d. $\int 2x(x^2+1)^5 dx$

2.   (10 pts.)   What is the average value of $f(x)=\sin(x)$ on the interval $[0,\pi]$?

3.   (10 pts.)   Evaluate

$$\frac{d}{dx}\int_{1}^{x}\frac{1}{t}dt$$