# Math 180 F

# FINAL EXAM

NAME_____

Monday, December 15

200 pts.

## I. Some basic function theory

1. (5 pts.) Define the **composition** $f \circ g$ of functions f and g.

2. (5 pts.) Define the **inverse** of a function f(x)

3. (5 pts.). Suppose f(x) = 2x - 4. Find the inverse of f.

## II. Limits and continuity

1. (5 pts.) Define formally what we mean when we say that $\lim_{x \to a} f(x) = L$.

2. (10 pts.) How would you explain $\lim_{x \to a} f(x) = L$ to an intelligent friend who has not taken calculus? This is a (brief) essay question and your grade on this problem will depend on both the clarity and completeness (without going overboard) of your response (no more than what fits on the rest of this page)

3. (5 pts.) State the **intermediate value theorem**.

4. (10 pts.) We would like to approximate the square root of 5 by solving the equation $x^2 - 5 = 0$.

   a. There is a solution between x=0 and x=4. How does the intermediate value theorem tell us this is so?

   b. The method of bisection reduces the interval in which a solution is certain to be found to a shorter interval in which the solution is guaranteed to be found. In this case, we move from [0, 4] to another interval. What is that interval?

## III. Differentiation

1. (10 pts.) Let f(x) be a function. Formally define the **derivative** $f'(x_0)$ at a point $x_0$.

2. (5 pts.) What is the geometrical interpretation of a derivative

3, (5 pts.) Explain what a derivative is in terms of the rate of change of a given quantity.

4. (5 pts. each) Find the derivatives of the following functions:

a. $x^5 - 5x^2 + 7x + 47$

b. $(x^5 - 5x^2 + 7x + 47)(x^7 + 2x^4 - 2x + 5)$

c. $\dfrac{(x^5 - 5x^2 + 7x + 47)}{(x^7 + 2x^4 - 2x + 5)}$

d. $(x^5 - 5x^2 + 7x + 47)^{21}$

Problem III.4 continued

e. $e^{(x^2)}$

f. $e^{2x}(\sin(4x+3)+\cos(2x))$

IV. Some applications of derivatives

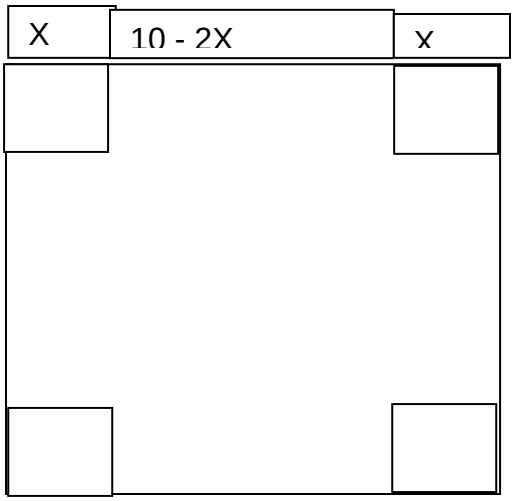
1. (10 pts.)

a. What is a critical point?

b. Identify the critical points of $f(x) = 2x^3 - 3x^2 - 12x + 30$

2. (10 pts.) Find the equation of the line tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ at the point $(\frac{3}{\sqrt{2}},\frac{4}{\sqrt{2}})$ (Please note that your answer is likely to have the radical $\sqrt{2}$ in it - please leave it in that form.)

3. (10 pts.) An open box is to be made from a tin sheet 10" square by cutting out squares of equal size on each corner and bending up the sides thus produced. Express the volume as a function of x. See the (attempted) diagram



4. (10 pts.) (15 pts.) (From Strauss, Bradley, and Smith Calculus) Suppose that it costs us $C(x) = \frac{1}{8}x^2 + 4x + 200$ dollars to manufacture and distribute **x** units of some commodity, and that we can sell each one for a price of (49-x) dollars per unit for a total revenue R(x) = $x(49 - x)$ dollars for x units. Our profit is then P(x) = R(x) - C(x). For what value of x will we obtain the largest profit?

IV Integration

1. (5 pts.) What is a **partition** of an interval? Define and give an example of a partition of the interval [0,2] with three intervals. Please look at the next two problems (on the next page) as you will use your solution to this problem for problems 2 - 4.

2. (5 pts.) What is the **norm** of a partition (of an interval)? Define and say what the norm of the partition you gave in problem 1 is.

3. (5 pts.) Pick a series $c_k$ of numbers with each $c_k \in [x_{k-1}, x_k]$

4 (10 pts.) What is a Riemann sum? Define, and give an example of a Riemann sum using the function $f(x) = x^2$ and the partition you specified in problem 1. You do not need to calculate the value of this particular Riemann sum.

5. (10 pts.) Define formally $\int_a^b f(x)dx$

6. (10 pts.) Suppose that F(x) is defined as $F(x) = \int_1^x \frac{1}{t} dt$. Find F'(x) (the derivative of F with respect to x. What important function do you think that F(x0 might be?

7. (5 pts. each) Solve the following integration problems. Partial credit for definite integral problems would be to simply find an antiderivative.

a. $\int_0^1 (x^5 - 5x^2 + 7x + 47)dx$

b. $\int_0^1 \frac{dx}{x}$

c. $\int_0^{\frac{\pi}{2}} \cos(x)dx$

6. (5 pts.) What is the average value of $f(x) = x^2$ on the interval [0,2]?