Math 180

FOURTH HOUR EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, Dec. 5, 2006
100 pts

I. Graphing

1. Consider the function $f(x) = x^3 - 12x$

a. (15 pts.) Find the critical points and note over which interval(s) the graph is concave up and concave down. Then use the second derivative test and concavity (show work) to characterize the critical points as local maxima, local minima, or points of inflection.

Critical Points:

Concave Up

Concave Down

(problem 1 continued)

b. (10 pts.) For this function, specify the intervals over which this function is **increasing** and the intervals over which it is **decreasing**. Show your work.

c. (5 pts.) Sketch the graph of this function

2. (10 pts.) Use **l'Hôpital's rule** to find the following limits. Show your work:

$$\lim_{x\to\infty}\frac{3x^2+2x-1}{6x^2-17x+3}$$

$$\lim_{x\to 0}\frac{\sin(3x)}{4x}$$

3. (10 pts.) Suppose that we want to solve the equation $x^2+x-6=0$. We use Newton's method with an initial guess of 3. What is the next guess?

4. (5 pts each) Find the following antiderivatives. Remember the constant of integration!

$$\int (3x^2 + 2x - 17)dx$$

$$\int e^{2x}dx$$

$$\int \sec(x)\tan(x)dx$$

$$\int \frac{dx}{1+x^2}$$

5. (10 pts.) (Problem 8 on page 277) A 216 square meter plot is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed? (be sure to draw a picture and label things).

6. (10 pts.) (Problem 48 on page 281). suppose that the cost of manufacturing x items is given by $c(x) = x^3 - 20x^2 + 20000x$. For what value of x do we minimize the **average** cost per item $(\frac{c(x)}{x})$

7. (10 pts.) Using the summation rules we have learned so far, calculate

$$\sum_{k=1}^{100} (3x - 2)$$