# Math 180 F

# FINAL EXAM

NAME________________________________________________________________________

Monday, December 17

200 pts.

## I. Definitions, theorems, and the like

1. (10 pts.) Define formally what we mean when we say that $\lim_{x \to a} f(x) = L$.

2. (5 pts.) Define formally what it means to say that f(x) is **continuous** at x = a.

3. (5 pts.) State the **intermediate value theorem**.

4. (10 pts.) Let f(x) be a function. Formally define the **derivative** $f'(x_0)$ at a point $x_0$.

5. (5 pts.) State the **mean value theorem for differentiable functions**, including what must be true for the theorem to be applied.

6. (5 pts.) What is a **partition** of an interval? Define and give an example of a partition of the interval [0,2]. Please look at the next two problems (on the next page) as you will use your solution to this problem for problems 7 and 8.

7. (5 pts.) What is the **norm** of a partition (of an interval)? Define and say what the norm of the partition you gave in problem 7 is.

8. (10 pts.) What is a Riemann sum? Define, and give an example of a Riemann sum using the function $f(x) = x^2$ and the partition you specified in problem 6.

9. (10 pts.) Define formally $\int_a^b f(x)dx$

10. (5 pts.) State one of the two fundamental theorems of calculus discussed in chapter 5.

II. Basic problems

1. Find derivatives of the following functions (with respect to x) (5 pts. each)

$$x^3 + 2x^2 + 17x - 22$$

$$(x^3 + 2x^2 + 17x - 22)(x^2 + 2x - 2)$$

$$\frac{(x^3 + 2x^2 + 17x - 22)}{(x^2 + 2x - 2)}$$

$$(x^3 + 2x^2 + 17x - 22)^{10}$$

Problem II.1 continued: Find derivatives of

$\cos(x)$

arctan(x)

$e^{(x^2)}$

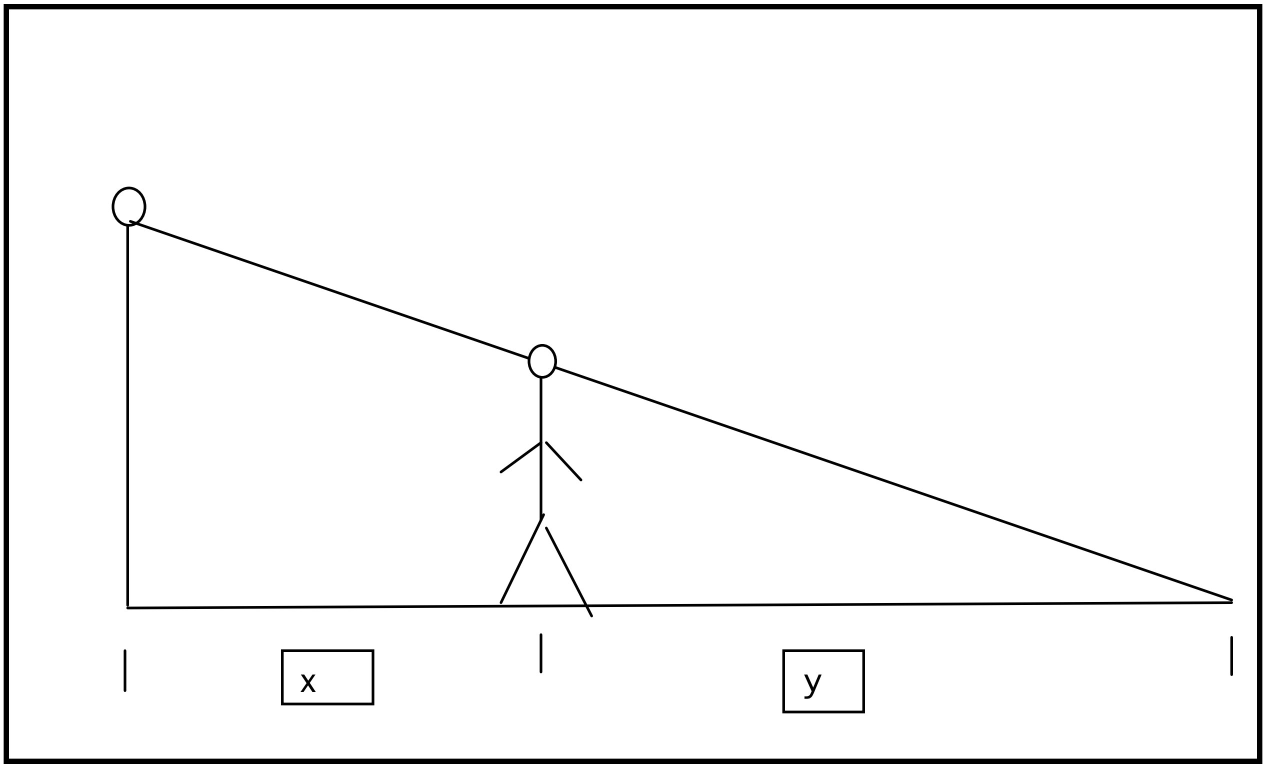
sinh(x)

y = (x-1)(x-2)(x-3) use logarithmetic differentiation.

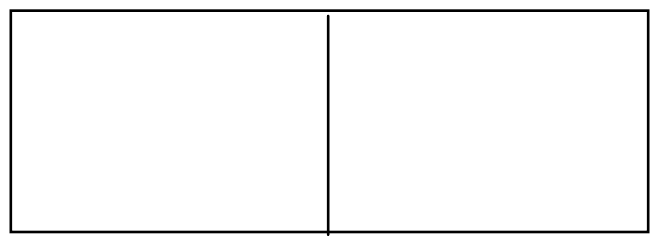
2. (10 pts.) Given the equation $x^2 + xy + y^2 = 7$, find y' = $\frac{dy}{dx}$ at the point (1,2) (x = 1, y = 2).

3. Applications (15 pts. each)

a. A person 6' tall walks away from a streetlamp 10' tall at the rate of 4 ft/sec, casting a shadow. How fast is the length of the shadow lengthening when the person is 6' from the lamp? Note that because of similar triangles, $\frac{x+y}{10} = \frac{y}{6}$



b.   We have 600' of fencing with which to enclose a rectangular area with a fence running down the middle.  What are the dimensions of the rectangle which maximizes the area of the enclosed area?



4. (15 pts.) Find critical points and asymptotes (including oblique) of $\frac{x^2}{x-1}$ together with regions in which the function is increasing or decreasing. Finally, briefly sketch the graph of the function.

III. Integration and the like

1. (5 pts. each) Solve the following integration problems. Be sure to remember the constant of integration for integrals without limits. Partial credit for definite integral problems: find an antiderivative.

$$\int_0^1 (x^3 + 2x^2 + 17x - 22)dx$$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$\int \sec^2(x)dx$$

$$\int_0^1 (x^3 + 2x^2 + 17x - 22)dx$$

(problem III.1 continued)

$$\int_1^e \frac{1}{x+1}\,dx$$

2. (5 pts.) What is the average value of $f(x) = x^2$ on the interval [0,2]?