Math 180 F

FOURTH HOUR EXAM

NAME____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, Dec. 7, 2007
100 pts

I. Graphing

1. Consider the function $f(x) = 2x^3 + 3x^2 - 12x$

a. (10 pts.) Find the critical points and use the second derivative test (show work) to characterize them as local maxima, local minima, or points of inflection.

b. (10 pts.) For this function, specify the intervals over which this function is **increasing** and the intervals over which it is **decreasing**. Show your work.

c. (5 pts.) Sketch the graph of this function. The original function crosses the x-axis at (roughly) -3.3, 0, and 1.8.

2. (5 pts.) Use **l'Hôpital's rule** to find the following limits. Show your work:

$$\lim_{x\to\infty}\frac{2x^2-3x+1}{4x^2-17x+15}$$

3. (10 pts.) Suppose that we want to solve the equation $x^2+x-1=0$. We use Newton's method with an initial guess of 2. What is the next guess?

4. (5 pts each) Find the following antiderivatives. Remember the constant of integration!

$$\int (x^3 + 3x^2 - 7x + 1)dx$$

$$\int \sin(x)dx$$

$$\int e^x dx$$

$$\int \sec(x)\tan(x)dx$$

$$\int \frac{dx}{1+x^2}$$

5. (15 pts.) (Problem #4 on page 276 of the textbook). A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions? Begin by drawing a picture of the situation.

6. (10 pts.) Evaluate $\sum_{k=1}^{5}(k+1)$ to a number

7. (10 pts.) Suppose we wish to find the area under the curve $y = f(x) = x^2$ between x=0 and x=1. To this end we divide the interval [0,1] into n equal sub-intervals, each of length 1/n, and pick the value of the function at the right-hand endpoint of each subinterval $(\{\frac{1}{n},\frac{2}{n},\frac{3}{n}\ldots,\frac{n}{n}\})$ for the height of a rectangle. We can then approximate the area under the curve as the sum

$$(\frac{1}{n})f(\frac{1}{n}) + (\frac{1}{n})f(\frac{2}{n}) + \ldots + (\frac{1}{n})f(\frac{n}{n}) = (\frac{1}{n})(\frac{1}{n})^2 + (\frac{1}{n})(\frac{2}{n})^2 + \ldots (\frac{1}{n})(\frac{n}{n})^2 = \sum_{k=1}^{n}(\frac{1}{n})(\frac{k}{n})^2 = (\frac{1}{n^3})\sum_{k=1}^{n}k^2$$

a. Using the formula for $\sum_{k=1}^{n}k^2$, write this sum as a function of n. You need only look at the right-most term in the above, the first part is just to remind ourselves how we got there.

b. If we let n go to infinity, to what number does the sum go to (as a limit)? Give your answer as a number