Math 180 C

FOURTH HOUR EXAM

NAME_____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, Dec. 1, 2006
100 pts

I. Graphing

1. Consider the function $f(x) = x(x^2 - 1)$

a. (10 pts.) Find the critical points and use the second derivative test (show work) to characterize them as local maxima, local minima, or points of inflection.

b. (10 pts.) For this function, specify the intervals over which this function is **increasing** and the intervals over which it is **decreasing**. Show your work.

c. (5 pts.) Sketch the graph of this function

2. (10 pts.) Use **l'Hôpital's rule** to find the following limits. Show your work:

$$\lim_{x \to 0} \frac{\sin(2x)}{3x}$$

$$\lim_{x \to \infty} \frac{\ln(x)}{x}$$

3. (10 pts.) Suppose that we want to find a cube root of 2 by solving the equation $x^3 - 2 = 0$. We use Newton's method with an initial guess of 2. What is the next guess?

4. (5 pts each) Find the following antiderivatives

$\int (x^2 + 3x - 7)dx$

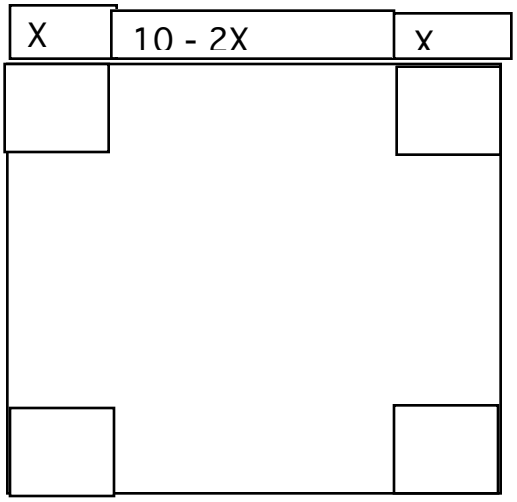
$\int \cos(x)dx$

$\int e^x dx$

$\int \sec^2(x)dx$

$\int \frac{dx}{1+x^2}$

5. (15 pts.) An open box is to be made from a tin sheet 10" square by cutting out squares of equal size on each corner and bending up the sides thus produced. See the (attempted) diagram



Express the volume as a function of x. For what value of x ($0 \le x \le 5$) will the volume be the largest possible?

6. (15 pts.) (From Strauss, Bradley, and Smith Calculus) Suppose that it costs us $C(x) = \frac{1}{8}x^2 + 4x + 200$ dollars to manufacture and distribute **x** units of some commodity, and that we can sell each one for a price of (49-x) dollars per unit for a total revenue R(x) $= x(49 - x)$ dollars for x units. Our profit is then P(x) = R(x) - C(x). For what value of x will we obtain the largest profit?