Hon 213

Third Hour Exam

Name ______________________________

Friday, April 27, 2007

1. (5 pts.) Some definitions and statements of theorems (5 pts. each)

   a, What is a Saccheri quadrilateral?

   b. State the Hilbert Parallel Postulate (being careful not to say more than you need to)

   c. State the Hyperbolic axiom (again being careful not to say more than you need to)

2. (15 pts.) Pick one of the equivalents to the parallel postulate given in propositions 4.7 - 4.11 (which may be found at the end of the exam) and provide a proof.

3. (5 pts.)

4. (10 pts.)

5. (5 pts.)

The story so far: Although there were many attempts to prove the parallel postulate, we will focus our attention here on two who almost had it figured out, and two who figured it out but whose contributions were not recognized until after their deaths, and (finally) consider the ghostly presence of one of the great names of mathematics.

6. (10 pts) Girolamo Saccheri (1667-1733) published *Euclid Freed of Every Blemish*. One of the blemishes was, of course, the parallel postulate. Briefly describe his approach, including a discussion of the three cases he considered for the quadrilaterals named after him. How did he handle the "acute hypothesis"?

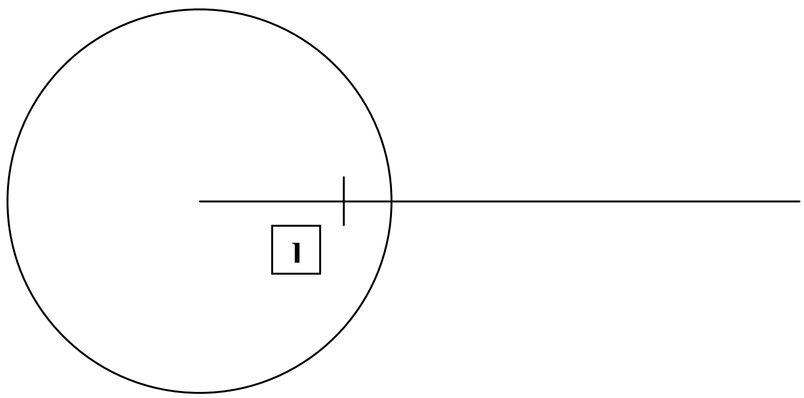
7. (10 pts.) What was Clairaut's axiom? Describe (briefly) how we can show it equivalent to the parallel postulate. Whose (failed) proof are we using?

Two mathematicians of the very first rank who almost had hyperbolic geometry in their hands. We next consider two who "got it", but were not very successful in getting the story out in their lifetimes.

9. (10 pts.) Briefly describe the efforts of János Bolyai (1802 - 1860) and Nikolai Lobachevsky (1792 - 1856) in developing what we now call hyperbolic geometry. How did their approach differ from that of Saccheri and Lambert? How were their ideas received?

10. (10 pts.) What role did the great Carl Friedrich Gauss (1777 - 1855) play in this with respect to the work of Bolyai and Lobachevsky? What role did he (Gauss) play in the development of non-Euclidean geometry?

11. (10 pts.) With straight-edge and compass construct the inverse of the point P in the circle below. Be sure to indicate marks so that I can see what you are doing. The point P below is indicated by a vertical bar '|' through a line which conveniently goes from the center of the circle, through P (which lies on the line) to a point outside the circle.



Random comment (more details on Friday):

Who made me the genius I am today,
The mathematician that others all quote?
Who's the professor that made me that way,
The greatest that ever got chalk on his coat?

One man deserves the credit,
One man deserves the blame,
and Nicolai Ivanovich Lobachevsky is his name. (Hey!)

-Tom Lehrer

Logic, axioms and selected major propositions

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

LR1: The following are the six types of justifications allowed for statements in proofs:
- (1) By hypothesis (given)
- (2) By axiom/postulate ...
- (3) By theorem ... (previously proved)
- (4) By definition ...
- (5) By step ... (a previous step in the argument
- (6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9:
- (a) []P => Q] & [Q => R]] => [P => R].
- (b) [P & Q] => P, [P & Q] => Q.
- (c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at lease two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A

BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.

BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.

BA4: For every line l and for any three points A, B, and C not lying on l,

1. if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l
2. If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l

   (corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l

CA1: Copying segments onto rays

CA2: Congruency of segments is an equivalence relation

CA3: Addition of segments

CA4: Copying angles

CA5: Congruency of angles is an equivalence relation

CA6: SAS

corr: copying triangles

Theorem 4.5: Euclid's fifth postulate if and only if Hilbert's parallel postulate.

Proposition 4.7: Hilbert's parallel postulate if and only if if a line intersects one of two parallel lines, then it also intersects the other.

Proposition 4.8: Hilbert's parallel postulate if and only if converse to theorem 4.1 (alt int angles)

Prop. 4.9: Hilbert's parallel postulate if and only if if T is a transversal to L and M, L is parallel to M, and T is perpendicular to L, then T is perpendicular to M.

Prop. 4.10: Hilbert's parallel postulate if and only if if K is parallel to L, M is perpendicular to K, and N is perpendicular to L, then either M = N, or M is parallel to N.

Prop 4.11: Hilbert's parallel postulate implies that the angle sum of every triangle is 180 degrees.