Honors 213

First Hour Exam

Name ________________________________

Friday, Feb. 16
100 points

I. Some definitions (5 points each). Give formal definitions of the following.

a. A segment

b. A right angle

c. A model

d. An affine plane

e. An isomorphism of models

f. What do models tell us about the relationship between the parallel postulate and the axioms of incidence geometry?

II. (10 pts.) Below draw a (straight) line and select some point on the line. Then using straight-edge and compass only, draw a line through the given point perpendicular to the line. Show all marks (if I can't see how you did it, you may not get credit for it).

III. (10 pts.) Fill in the gaps in the following proof. Logic rules and the axioms of incidence geometry are given on the last page of this exam.

Given: A point P
To show: That there is a line not passing through it.

Number	Statement	Justification
1	Let P be a point	
2.	Suppose that all lines pass through P	
3.	There exist three distinct lines that are not concurrent; call them l, m, and n	Prop. 2.2
4.	l, m, and n do not pass through any given point	
5.	l, m, and n pass through P	
6.	Therefore the assumption in 2 is false, and there is a line not passing through P.	

III. (5 pts. each). Given the statement "If it is raining I get wet". Give, in ordinary English:

a. The converse to the statement

b. The contrapositive of the statement

c. What is the sufficient part?

d. What is the necessary part?

e. How does the mathematical interpretation of this statement differ from the natural language interpretation of this statement?

V. (5 pts. each) Writing the negation of a statement is a useful thing to be able to do. In part (a), write down the negation of the statement, simplifying it so that negations occur only before predicates. In part (b), state the negation in conversational English.

a. $\exists x(p(x) \wedge \neg q(x))$

b. All right angles are congruent

VI (10 pts.) One of the rules of logic asserts that not (p or q) is equivalent to (not p) and (not q).

$$\neg(p \Rightarrow q) \equiv (p \wedge \neg q)$$

Use truth-tables to demonstrate this (the following diagram should get you started). What in the completed truth table tells you that you should believe the statement?

p	q	p or q	not (p or q)	not q	not p	(not p) and (not q)

VII (5 pts.) Say something (a brief sentence - something relevant to the course) about three of the following names.

a. Euclid
b. Thales of Miletus
c. David Hilbert
d. Proclus Diadochus
e. Gottlob Frege
f. Adrien Marie Legendre
g. Pythagoras of Samos

LR0: No unstated assumptions.

LR1: The following are the six types of justifications allowed for statements in proofs:
(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at lease two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.
: For every point P there exist at least two lines through P

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A
BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.
BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.