CSci 370

Final Exam

Name _____

Friday, December 14
200 points.

I. Some basic models of computation: Finite Automata

1. (10 points) Give a definition of a Deterministic Finite Automaton

2. (10 points) Suppose that an **integer** can consist of an **optional** +/- sign followed by one or more digits followed by an (again optional) exponent in the form of an E followed by a two digit number. Construct a DFA which recognizes items of type integer. The language is Σ={+, -, E, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Please feel free to let the regular expression **digit** stand for the set of digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

3. DFA's are equivalent to **regular expressions**.

   a. (10 points) Give a definition of a regular expression.

   b. (5 points) Give the regular expression for the DFA constructed in problem 2 above (i.e., a regular expression for the integer type given in problem 2).

4. (5 points) Briefly, what role do deterministic finite automata play in compilation?

5. (5 points) What does it mean to say that a DFA D **recognizes** a language L?

6. (5 points) What is a **regular language**? Why is it called that?

7. (15 points) The **Pumping Lemma** for deterministic finite automata tells us that not all languages are regular.

a. State the Pumping Lemma for deterministic finite automata.

b. The language $L = \{0^n 1 0^n \mid n \geq 0\}$ is not regular. To use the pumping lemma, what string in the language would you use to show this? That is, give a string in the language which can not be "pumped".

c. Give a brief argument saying why the string you chose can not be "pumped".

II. Some basic models of computation: Push-down automata

1. (10 points) Give a definition of a push-down automaton.

2. (10 points) Describe (using the graphical notation of our textbook) a (deterministic) PDA which recognizes the language $L = \{0^n 1 0^n \mid n \geq 0\}$

3. (10 points) Give a definition of a context-free grammar.

4. (10 points) Give a context-free grammar for the language $L = \{0^n 1 0^n \mid n \geq 0\}$.

5. Although a language recognized by a deterministic finite automata is also recognized by a push-down automata (and is thereby a context free language), the reverse is not true.

   a. (10 points) State the pumping lemma for Context Free Languages.

   b. (5 points) Give an example of a language that is not context free. You do not need to demonstrate that the language you specify is not a CFL.

6. (5 points) What is the relationship between push-down automata and the compilation process?

III. Some basic models of computation: Turing Machines.

1. (10 points) What is a Turing Machine (give a formal definition).

2. (10 points) What does it mean when we say that a language L is
   a. Turing Recognizable?

   b. Decidable?

3. (10 points) Briefly outline (in words) a Turing Machine which recognizes the language $L = \{0^n 1 0^n \mid n \geq 0\}$

IV. Time Complexity of decidable languages

1. (10 points) Define the class P and give an example of a language in P.

2. (5 points) What is a **verifier** in the context of the class NP?

3. (5 points) Define the class NP and give an example of a language in NP not known to be in P

4. (5 points)Define the class NP-Complete, and give an example of an NP-Complete language.

V. (20 points) An essay question. Pick one of the three problems below and write a brief essay on it. Do not do both – only the first one you give will be graded. Please put your answer on the following page.

a. One of the main themes of this course is the question: "What is computable?" In search of an answer to that question we have examined several different models for computation, ending with the Turing Machine. The Turing-Church Hypothesis is (I think) our best answer so far, but you should certainly feel free (perhaps obligated) to disagree. What is the Turing-Church hypothesis, and explain why you might believe it true or untrue.

b. During the course of the term we have discussed several models for computation. These form a part of what we call the Chomsky Hierarchy. Discuss the Chomsky Hierarchy, and how the classes of languages we have discussed relate to each other in that hierarchy. There are lots of gaps in the hierarchy that we have not discussed. Can you think of one?

c. There have been two questions guiding our discussion this term: What is computable, and what is "reasonably" computable. In the final weeks of the term we have briefly discussed the second part of this question. Discuss the notions of P, NP, and NP-Completeness. Why is this final question important as well?

(page provided for your answer to question V)