## CSci 370

## First Hour Exam

## Name

Friday, Sept. 28
100 points.
I. Some definitions. Give formal definitions of the following.

1. (5 points) Deterministic Finite Automata (DFA)
2. (5 points) Nondeterministic Finite Automata (NFA)
3. (5 points) Regular Language
4. (10 points) What does it mean (formally) for a DFA to recognize a string over the alphabet of the DFA?
II. Just checking (15 points)

Suppose that sets $A=\{a, b, c, d\}$ and $B=\{b, e\}$. What is

1. $A \cup B=$
2. $A \cap B=$
3. $A X B=$
III. (10 points) An induction question. Suppose that we want to show that $\sum_{k=1}^{n}(2 k-1)=n^{2}$ (the sum of the first $n$ odd numbers is $n^{2}$ ) using mathematical induction.
4. State and prove the base case
5. State and prove the inductive step. Identify the inductive hypothesis in your proof.
IV. A question on DFA's. Given a DFA over the alphabet $\Sigma=\{0,1\}$ with the following transition function:

|  | 0 | 1 |
| :--- | :--- | :--- |
| Q0 | Q0 | Q1 |
| Q1 | Q2 | Q1 |
| Q2 | Q0 | Q3 |
| Q3 | Q3 | Q3 |

with Q0 the start state and Q3 the only final state.

1. (10 points) Sketch the DFA
2. (5 points each) For each of the following two strings over $\Sigma$, list the states encountered by the DFA and say whether or not the string is accepted:
a. 011010
b. 010010
3. (5 points) What does this DFA do?
V. A question on NFA's. Given a NFA over $\Sigma$ with the following transition function:

|  | 0 | 1 | E |
| :--- | :--- | :--- | :--- |
| Q0 | $\}$ | $\}$ | $\{$ Q1. Q3 $\}$ |
| Q1 | $\{Q 1\}$ | $\{Q 1, \mathrm{Q} 2\}$ | $\}$ |
| Q2 | $\{Q 6\}$ | $\}$ | $\}$ |
| Q3 | $\{Q 3$, Q4\} | $\{Q 3\}$ | $\}$ |
| Q4 | $\}$ | $\{Q 5\}$ | $\}$ |
| Q5 | $\{Q 5\}$ | $\{Q 5\}$ | $\{Q 6\}$ |
| Q6 | $\{Q 6\}$ | $\{Q 6\}$ | $\}$ |

Where Q0 is the start state, and Q6 is the only final state. \{ \} indicates the empty set.

1. (5 points) Sketch this NFA. It should (sort of be the union of two NFA's: One consisting of Q0, Q1, Q2, and Q6, and the second consisting of Q0, Q3, Q4, Q5, and Q6.
2. (5 points) What language is recognized by the part consisting of Q0, Q1, Q2, and Q6? What regular expression describes this part?
3. (5 points each) In converting a NFA to a DFA, the states of the DFA are subsets of states of the NFA:
a. How many such states are possible? (note that there are 7 states in the NFA)
b. What state (subset of states) corresponds to the start state of the DFA?
c. What state results from the transition function on the state $\{\mathrm{Q} 4\}$ on input 1 ?
