Math 180 C

FINAL EXAM

NAME____

Friday, December 15

200 pts.

I. Definitions

1. (10 pts.) Define **formally** what we mean when we say that $\lim_{x \to a} f(x) = L$.

2. (5 pts.) Define **formally** what it means to say that f(x) is **continuous** at x = a.

3. (10 pts.) Let f(x) be a function. Formally define (limit definition) the **derivative** $f'(x_0)$ at a point x_0.

4. (5 pts.) What is a **partition** of an interval? Illustrate your definition by giving a partition of the interval [2,4].

5. (5 pts.) Give a example of a Riemann sum.

7. (5 pts.) Define formally $\int_a^b f(x)dx$ in terms of Riemann sums.

II. Theorems

1. (5 pts.) State the **intermediate value theorem**, including what must be true for the theorem to be applied.

2. (5 pts.) State the extreme value theorem for continuous functions.

3. (5 pts.) What are critical points, and why are they important to us?

4. (5 pts.) State the **mean value theorem.**.

5. (5 pts.) State one of the two fundamental theorems of calculus discussed in chapter 5.

II. Basic problems

1. (5 pts. each) Find derivatives of the following functions (with respect to x). Carry your calculations to the point that no more derivatives need to be taken.

$$x^7 - 3x^5 + 7 - 2x \quad + 3x - 17$$

$$(x^2 - 2x + 1)(7x^3 + 2x^2 + x - 1)$$

$$\frac{(x^2 - 2x + 1)}{(7x^3 + 2x^2 + x - 1)}$$

(Problem II.1 continued. Find derivatives of the following functions)

$(x^2 - 2x + 1)^{10}$

sec(x)

$e^{(x^2-1)}$

2. (10 pts.)

a. What is arctan(1)? Give as a number.

b. Write tan(arccos(x)) as an expression in x (but without trig or inverse trig functions)

3. (10 pts.) Suppose that the acceleration of a particle is given by $a(t) = -\sin(t)$, and that v(0) = 2, x(0) = 1. Find x(t) for the particle.

III. Applications

1. (10 pts.) Hook's law tells us that the force acting on a spring is F(x) = -**k**x, where x is the distance from an equilibrium position (x=0) and **k** is some constant. We'll take k = 5 for this problem. If we pull on the spring at the rate $\frac{dx}{dt} = 2$, what is $\frac{dF}{dt}$ when x = 1?

2. (15 pts.) The cost of producing x items is given by $C(x) = 2x^3 - 5x^2 + 1500x$. For what value of x is the average cost C(x)/x minimized?

3. (15 pts.) A rectangular box with square ends is to be shipped. Regulations require that the girth of the box (the perimeter of the square end plus the length, $(2x + l)$, where x is the length of one of the square ends and l is the length of the box, can be no more than 84 inches. We want to find the maximum possible volume of the box (i.e., when the girth is 84 inches).

a. Draw a picture, labeling all the features of the problem.

b. Find a formula giving the volume of the box in terms of the length of a side of a square end (x).

(problem II.4 continued)

c. Use that formula to find a possible value for x.

d. Use the second derivative test to see if your answer gives a maximum.

III. Integration and the like

1. (5 pts. each) Solve the following integration problems:

Indefinite integrals: Remember the constant of integration!

$$\int (x^2 - 1)dx$$

$$\int \cos(x)dx$$

$$\int \sec(x)\tan(x)dx$$

Definite integrals

$$\int_1^2 (x^2 + x + 1)dx$$

$$\int_0^{\frac{\pi}{2}} \cos(x)dx$$

(Definite integrals, continued)

$$\int_0^1 \frac{dx}{x^2+1}$$

2. (10 pts.) Using the rules of summations, calculate $\sum_{k=1}^{10} (3k-7)$