More problems on logic

Each of the following problems involves the definition of a type of function. Each definition is stated as it might appear in a typical calculus text. You will be rewriting the defining property using the quantifiers $\forall$ and $\exists$. Note that the quantification is not always explicit in the given definition. The universal set will be the domain of the function f in the statement. Use D to denote the domain of the function f.

1. Definition: A function f is *even* if $f(x) = f(-x)$ for all x in the domain of f.
   (a) Write the defining property as a quantified statement.
   (b) Prove that $f(x) = x^2$ is even.
   (c) Write the negation of the property that defines even.
   (d) Prove that $f(x) = x^3$ is not even.
2. Definition: A function f is *periodic* if for some number $p > 0$, $f(x + p) = f(x)$ for all x in the domain of f.
   (a) Write the defining property as a quantified statement.
   (b) Prove that $f(x) = \sin x$ is periodic. (You can assume that the reader knows all of the standard trigonometric identites.)
   (c) Write the negation of the property that defines periodic.
   (d) Prove that $f(x) = x$ is not periodic.
3. Definition: A function f is *decreasing* if $f(x) < f(y)$ whenever $x > y$.
   (a) Write the defining property as a quantified statement.
   (b) Prove that $f(x) = -x^3$ is decreasing.
   (c) Write the negation of the property that defines decreasing.
   (d) Prove that $f(x) = x^2$ is not decreasing.
4. Definition: A function f is *one-to-one* if $x = y$ whenever $f(x) = f(y)$.
   (a) Write the defining property as a quantified statement.
   (b) Prove that $f(x) = -x^3$ is one-to-one.
   (c) Write the negation of the property that defines one-to-one.
   (d) Prove that $f(x) = x^2$ is not one-to-one.