Problems on logic

1. For each of the following, determine the truth value of the given statement when p and q are true and r is false.

    (a) $\sim (p \Rightarrow q)$

    (b) $(\sim p) \Rightarrow r$

    (c) $p \Rightarrow (r \Rightarrow q)$

2. Suppose $p \Rightarrow q$ is false. Determine the truth value of $\sim (p \wedge q)) \Rightarrow q$.

3. For each of the following, choose specific statements for p, q, and r so that the given statement is true. Also choose a different set of specific statements for p, q, and r so that the given statement is false.

    (a) $(\sim (p \wedge q)) \Rightarrow r$

    (b) $r \Rightarrow (p \vee q)$

4. For each of the following, construct a truth table to show that the given statements are logically equivalent.

    (a) $\sim (p \vee q) \quad \longleftrightarrow \quad (\sim p) \wedge (\sim q)$

    (b) $p \Rightarrow q \quad \longleftrightarrow \quad \sim (p \wedge (\sim q))$

    (c) $p \vee q \quad \longleftrightarrow \quad (\sim p) \Rightarrow q$

5. A *tautology* is a statement that is true under all conditions. Construct a truth table to show that $p \vee (\sim p)$ is a tautology.

6. Construct a truth table to show that

$$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

is a tautology. Note that we did part of this in class but failed to consider all possible combinations of truth values for p, q, and r. You will need 8 rows in your truth table.

7. A *contradiction* is a statement that is false under all conditions. Construct a truth table to show that $p \wedge (\sim p)$ is a contradiction.