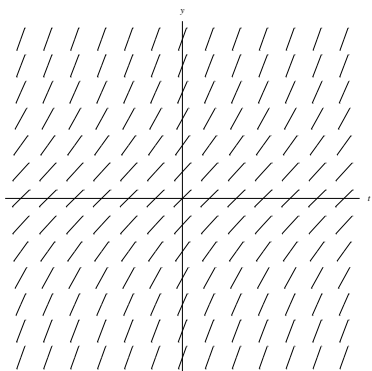


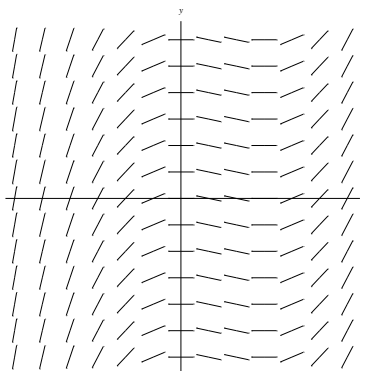
**Instructions:** You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

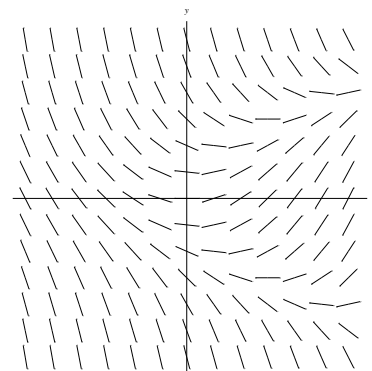
1. Below are three slope fields.



Slope Field A



Slope Field B



Slope Field C

- (a) Match each of these slope fields with one of the following: (6 points)

i.  $\frac{dy}{dt} = f(t)$

ii.  $\frac{dy}{dt} = f(y)$

iii.  $\frac{dy}{dt} = f(t, y)$

- (b) Which of the cases in (a) can be labeled *autonomous*? (3 points)

2. For each of the following, find all solutions of the given differential equation.

(a)  $\frac{dy}{dt} = t\sqrt{y}$  (15 points)

(b)  $\frac{dy}{dt} = \frac{4}{t}y + t^4e^{-2t}$  (for  $t > 0$ ) (15 points)

3. Consider the differential equation  $\frac{dy}{dt} = -\frac{y}{t} + 2$  for  $t > 0$ .

(a) Verify that  $y = C/t + t$  is a solution for any constant  $C$ . (5 points)

(b) Explain how  $y = C/t + t$  fits into our theory on the algebraic structure of solutions to linear first-order differential equations. (5 points)

(c) Find the specific solution that satisfies the initial condition  $y(1) = 0$ . (4 points)

4. Consider the equation  $\frac{dy}{dt} = 9y - y^3$ .

- (a) Find and classify (as sink, source, or node) all equilibrium points for this equation. (8 points)
- (b) Draw the phase line for this equation. (4 points)
- (c) Make a plot in the  $ty$ -plane showing solutions that represent each of the possible types of solution behavior. (4 points)

5. Consider the equation  $\frac{dy}{dt} = 40 - 10y^2$  with initial condition  $y(0) = 1$ .

- (a) Compute two steps of Euler's method using a step size  $\Delta t = 0.1$ . (8 points)
- (b) Use qualitative analysis to argue that the  $y$  values you compute in (a) are *not* good approximations for the corresponding  $y$  values from the exact solution. (6 points)

6. Consider some species in which adults generally live alone, perhaps some type of lizard in a particular desert. Let  $P$  measure the number of individuals in this population. In order to reproduce, a male and female must meet. It is reasonable to model the probability of a reproductive encounter as proportional to the product of the number of males and the number of females in the region. If the proportions of males and females are equal, this product is proportional to the square of the total population  $P^2$ . So, the probability of a reproductive encounter, and hence the birth rate, is proportional to  $P^2$ . The death rate is assumed to be proportional to  $P$ .

- (a) Write down the differential equation for  $P$  based on these assumptions about the birth and death rates. Name your own parameters and state any assumptions you are making about the values for each parameter. (5 points)
- (b) There is one positive equilibrium point for this model. Find this in terms of the parameters in your model. (4 points)
- (c) What does the model predict for a population that is below the equilibrium value in (b)? (4 points)
- (d) What does the model predict for a population that is above the equilibrium value in (b)? (4 points)