Name ______________________________

MATH 301 Differential Equations Spring 2011 Project #1

Instructions

Each project will consist of one or more problems or tasks. You can work on the details of problems with others. In fact, I encourage you to do so. Get a group of two or three people together, find a blackboard, and go to it.

For each project, you will submit a carefully written report on your results. All of your writing should be done independently even if you have worked on details with others. Your report should be self-contained so that a reader can understand the context and results without having read the problem statement.

For your writing, you should consider the audience to be familiar with the material we have seen so far in this differential equations course but who have not looked at the particular problems at hand. You should include enough detail so that a reader in this audience could follow your reasoning and reconstruct your work. In your writing, focus on being precise, concise, and clear.

You should write using the style and tips given on the handout "Notes on writing in mathematics". When appropriate, you should include carefully drawn or printed figures and plots. Since typesetting mathematics is difficult, you can write project reports neatly by hand. Another option is to use a word processor and then write mathematical expressions in by hand. You can also use an "equation editor" if one is available in your word processor but this can be time consuming so you need not do so.

The project is due in class on Monday, January 31.

Broadly speaking, there are three steps in modeling a real-world phenomenon with differential equations:

- formulate differential equations that reflect important aspects of the phenomenon;
- analyze the differential equations to determine solutions or properties of solutions; and
- compare results from the analysis with data from experiment or observation.

For this project, you will focus only on the first of these steps. A differential equations model is set up and analyzed in the accompanying paper "Reduction of HIV Concentration during Acute Infection: Independence from a Specific Immune Response", Andrew N. Phillips, *Science*, New Series, Vol. 271, No. 5248 (Jan. 26, 1996), pp. 497-499.

In the Phillips paper, the differential equations are labeled as Equations (1), (2), (3), and (4). **Your task is to understand the reasoning used to formulate these equations and to express that reasoning in your own terms.** The author's reasoning is given in the second paragraph of the paper.

The general idea behind this model is to think about some specific type of stuff in a container. (The container could be a real physical container or could be a conceptual container.) The amount of stuff in the container can change in time (by various processes). Let A represent some measurement of the amount of stuff. (For example, if the container is a tank and the stuff is water in the tank, we might measure the volume of the water or we might measure the mass of the water.) Since the amount of stuff in the container can change in time, we should think of A as a function of time t. The derivative dA/dt then represents the rate of change in the amount of stuff in the container. Each process contributes to the overall rate of change. So, if we identify independent processes, we can then write an equation in the form

$$\frac{dA}{dt} = \left(\begin{array}{c}\text{rate of change due to}\\ \text{first process}\end{array}\right) + \left(\begin{array}{c}\text{rate of change due to}\\ \text{second process}\end{array}\right) + \cdots + \left(\begin{array}{c}\text{rate of change due to}\\ \text{last process}\end{array}\right)$$

The rate of change due to each process might depend on A itself.

Equations (1)-(4) in the Phillips paper all have the general structure described in the previous paragraph. Your job is to understand and explain how these equations model the specific real-world phenomena in question. Here are some general questions that might guide your thinking:

- What is the container?
- What types of stuff in the container are being studied?
- For each type of stuff, what are the processes that change the amount of that stuff in the container?
- For each process, what is the corresponding term in the relevant differential equation?
- How does the form of each term relate to the process's contribution to the overall rate of change?