

### Exploring nonlinear systems

1. Start up the JODE applet for systems of equations. The fastest way to find it is to type “jode applet” into a search engine. The author’s version is at [math.arizona.edu](http://math.arizona.edu). There are also mirrors at other sites. You might want to change some of the default settings. Here’s some suggestions:

- Start by hitting the **Frame** button in the lower right corner of the applet window. This will “detach” the applet window from the web page. When you click on the detached window, you will get some extra menu items that give you some control on printing, saving, and loading.
- To reduce some of the clutter, uncheck the **Point** and **Init. Cond** options.
- To get more accurate approximate solution curves, change **Mod. Euler** to **RK4**. Later, you might want to also change the step size.

2. Use the applet to generate conjectures about the phase portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= x - x^3.\end{aligned}$$

Here are some ideas on things to do in exploring the phase portrait:

- Locate equilibrium points (either exactly using algebra or approximately from the tangent vector field). Look at a window that includes all equilibrium points. Also, zoom in on each equilibrium point.
- Try to classify solution curves into groups that have similar features. That is, generate conjectures of the form “All solution curves in Group A have Features X, Y, and Z.”
- Look for special solution curves that are distinguished by having features not shared by other solution curves. That is, generate conjectures of the form “There is a solution curve that has Features P, Q, and R.”

3. Use the applet and suggestions to generate conjectures about the phase portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= 2x\left(1 - \frac{x}{2}\right) - xy \\ \frac{dy}{dt} &= 3y\left(1 - \frac{y}{3}\right) - 2xy.\end{aligned}$$