Practice with judicious guessing

In the problems below, you will work with the differential equation

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = f(t)$$

for various choices of the nonhomogeneous term $f(t)$.

1. Solve the related homogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 0$.

2. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 6$.   Hint: Try $y_p = A$.

3. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 3t$.   Hint: Try $y_p = A + Bt$.

4. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = -2e^{2t}$.   Hint: Try $y_p = Ae^{2t}$.

5. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = e^{4t}$.   Hint: First try $y_p = Ae^{4t}$.
   When this doesn't work, articulate what is going on here. Then try $y_p = Ate^{4t}$.

6. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 6\cos(3t)$.   Come up with your own judicious guess.

7. Solve the nonhomogeneous problem $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 6e^{3it}$.   Come up with your own judicious guess.

8. From your particular solution in Problem 7, extract a particular solution for Problem 6.

9. From your particular solution in Problem 7, extract a particular solution for
   $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 6\sin(3t)$.