

Exam #2 objectives

Note: In the objectives below, “ 2×2 system” refers to a 2×2 system of first-order differential equations.

For Exam #2, a well-prepared student should be able to

- express a system of first-order differential equations in component form or in vector form
- understand the connections between a solution curve relating the dependent variables and the graphs of each dependent variable as a function of the independent variable
- state the Existence-Uniqueness Theorem and use this theorem to provide information on qualitative features of a system of first-order differential equations
- construct and interpret a tangent or direction vector field for an autonomous 2×2 system
- use a tangent or direction vector field to sketch an approximate phase portrait
- find the equilibrium points for an autonomous system of first-order differential equations
- convert a second-order differential equation into a system of first-order differential equations
- solve a decoupled or partially decoupled system of first-order differential equations
- use Euler’s method to find an approximate solution for a system of first-order differential equations with initial condition
- recall and use relevant facts from linear algebra
- express a linear system in matrix form
- determine whether or not a linear system of differential equations has a unique equilibrium point
- describe and use the structure of the general solution for a linear homogeneous 2×2 system
- construct the general solution for a linear homogeneous autonomous 2×2 system in the cases of distinct nonzero real eigenvalues and complex eigenvalues
- use the general solution to find a specific solution for a linear homogeneous autonomous 2×2 system and initial condition
- draw a quantitatively accurate phase portrait for a linear homogeneous autonomous 2×2 system with distinct real eigenvalues
- draw a qualitatively accurate phase portrait for a linear homogeneous autonomous 2×2 system with complex eigenvalues
- construct the general solution for a linear homogeneous autonomous 2×2 system with repeated eigenvalue or zero as an eigenvalue
- build, interpret, and analyze simple differential equation models for real-world phenomena such as predator-prey interactions and the motion of an object on a spring