Name ______________________________

MATH 280 Multivariate Calculus Spring 2007 Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

Note: Throughout this exam, x, y, and z refer to cartesian coordinates; r and θ refer to polar coordinates (so r, θ and z refer to cylindrical coordinates); and ρ, ϕ, and θ refer to spherical coordinates.

1. Give the name of the Greek letter ρ. ____________ (1 points)

2. For each of the following, give a brief answer. Aim for an audience of students in another section of multivariate calculus who have not yet studied Chapter 13.

   (a) What is a *double integral*? (3 points)

   (b) What is an *iterated integral in two variables*? (3 points)

   (c) What does Fubini's Theorem tell us about the relationship between the double integral $\iint\limits_R f\,dA$ and the iterated integral $\int_a^b \int_c^d f(x,y)\,dy\,dx$ if the region R is described by $a \le x \le b$ and $c \le y \le d$ and the function f is continuous for R? (3 points)

3. Give a geometric justification for the form of the area element in polar coordinates. That is, explain how to get the formula $dA = r\,dr\,d\theta$. Include a relevant picture. (8 points)

4. Evaluate the double integral $\iint\limits_R f\,dA$ where $f(x,y) = x^2 + xy$ and R is the region in the first quadrant of the xy-plane bounded by $y = 1$, $x = 5$, and $y = x^3$. You can stop when only arithmetic remains. (12 points)

5. Set up an iterated integral or integrals to compute the area of one "petal" of the polar curve given by $r = 7\cos(5\theta)$. Express your result entirely in terms of a single coordinate system (of your choice). You do not need to evaluate the integral or integrals. (10 points)

6. Set up an iterated integral or integrals equal to the double integral $\iint\limits_R f\,dA$ where $f(x,y) = x + y$ and R is the half of the disk $x^2 + y^2 \le 9$ with $y \le 0$ using

   (a) cartesian coordinates (10 points)

   (b) polar coordinates (10 points)

   You do not need to evaluate any of these integrals.

There is more on the flip side.

7. Charge is distributed on a disk of radius R. The area charge density is proportional to the distance from the center of the disk and has a value σ_0 on the edge of the disk. Compute the total charge on the disk in terms of σ_0 and R. (10 points)

8. Set up an iterated integral or integrals equal to the volume of the solid region in the first octant of a cartesian coordinate system bounded by the coordinate planes, the plane $2x + 3y = 6$, and the cylinder $z = 4 - y^2$. Express your result entirely in terms of a single coordinate system (of your choice). You do not need to evaluate the integral or integrals. (10 points)

9. Set up an iterated integral or integrals in spherical coordinates equal to the triple integral $\iiint\limits_D f\,dV$ where $f(x, y, z) = x$ and D is the portion of the solid sphere $x^2 + y^2 + z^2 \leq 9$ that lies in the first octant. You do not need to evaluate the integral or integrals. (10 points)

10. Charge is distributed in a solid right circular cylinder of radius R and height H. The volume charge density is proportional to the distance from one end of the cylinder. Compute the total charge on the region in terms of R, H, and a proportionality constant you name. (10 points)