Name ____________________

MATH 280 | Multivariate Calculus | Spring 2007 | Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

---

1. Give a parametric description for the line that contains the point $(3, -4, -2)$ and is perpendicular to the plane given by $5x + y + 2z = 6$. Give both a vector equation and separate scalar equations. (10 points)

2. A piece of wire is wrapped around the elliptic cylinder given by the equation $\frac{1}{9}x^2 + y^2 = 1$ to form an elliptic helix. The wire wraps counterclockwise from the viewpoint of looking down the z-axis. For each wrap around the cylinder, the helix rises 3 units in the z-direction. Find a parametrization for four wraps of this elliptic helix starting from the point $(3, 0, 0)$. Give a range of values for your parameter. (8 points)

3. Consider the surface given by the equation $\frac{1}{4}x^2 - \frac{1}{25}y^2 - z^2 = 1$.

   (a) Sketch the cross-sections of this surface in the xy-plane, the xz-plane, and the yz-plane. (9 points)

   (b) Use words and/or pictures to describe the surface. (5 points)

4. Think of a vector output function $\vec{f} : \mathbb{R} \to \mathbb{R}^3$ as tracing out a curve in space.

   (a) State a *geometric* interpretation for an output of the derivative $\vec{f}\,'(t)$ in relation to the curve traced out by $\vec{f}$. (5 points)

   (b) Justify the interpretation you give in (a) starting from the definition of the derivative $\vec{f}\,'$. (5 points)

5. Consider the curve traced out by $\vec{r}(t) = 5t^2\,\hat{\imath} + 6t\,\hat{\jmath} + (1 + t^3)\,\hat{k}$ and the plane given by $x - 2y + z = 5$.

   (a) Show that the curve intersects the plane for $t = 2$. (4 points)

   (b) Find a vector tangent to the curve at the point of intersection corresponding to $t = 2$. (6 points)

   (c) Find the angle between the tangent vector from (b) and a normal vector for the plane. (4 points)

There is more on the flip side.

6. A projectile is launched in a perfectly flat area on a windy day. During the projectile's flight, a gust of wind provides a constant horizontal acceleration of 4 $\mathrm{m/s^2}$ in what we will choose as the $\hat{\imath}$ direction. Gravity also provides a constant downward acceleration of 10 $\mathrm{m/s^2}$ (rounded for convenience). We will choose $\hat{k}$ in the upward direction. The total acceleration is thus $2\,\hat{\imath} - 10\,\hat{k}$ $\mathrm{m/s^2}$. We can choose the origin to be the point from which the projectile is launched. Suppose the initial velocity is $5\,\hat{\jmath} + 30\,\hat{k}$ m/s.

   (a) Find the velocity function $\vec{v}(t)$ for the object. (5 points)

   (b) Find the position function $\vec{r}(t)$ for the object. (5 points)

   (c) Find the point at which the projectile hits the ground. (3 points)

   (d) Set up an integral to give the total length of the projectile's path from launch to impact. You do not need to evaluate the integral. (4 points)

7. Consider the function $f(x, y) = \dfrac{y}{x^2}$.

   (a) Determine the domain and range of this function. (4 points)

   (b) Sketch one plot showing level curves for the outputs $-2, -1, 0, 1, 2$. (7 points)

   (c) Use words and/or pictures to describe the graph of this function. (4 points)

8. For each of the following, either determine the limit or show that the limit does not exist. (6 points each)

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4 + y^4}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x + y}{x^2 - 5x + xy - 5y}$