Name ____________________

MATH 280 Multivariate Calculus Fall 2010 Exam #4

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

A. Throughout this exam, x, y, and z refer to cartesian coordinates; r and θ refer to polar coordinates (so r, θ and z refer to cylindrical coordinates); and ρ, ϕ, and θ refer to spherical coordinates as we have defined these in class.

B. Pay attention to the statement of each problem. If a problem asks you to "Set up an iterated integral or integrals…" you do not need to evaluate the integral(s).

C. For full credit, each iterated integral you set up should be expressed entirely in terms of one coordinate system.

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1. Provide a brief answer to each of the following questions.
   (a) What, at a fundamental level, is integration? (6 points)
   (b) What is the essential distinction between a *double integral* and a *triple integral*? (6 points)

2. Give an exact geometric description of the curve given by the polar equation $r = 6\sin\theta$. Show or explain how you reach your conclusion. (7 points)

3. After peeling an orange, you find it is a perfect sphere of radius 5 cm and that it has 10 perfect slices, all of equal size. Describe one slice in a coordinate system of your choice. Include a description or a sketch of how you set up your coordinate system. (8 points)

4. Give a geometric justification for the form of the area element in polar coordinates. That is, explain how to get the formula $dA = r\,dr\,d\theta$. Include a relevant picture. (9 points)

5. Use polar coordinates to set up an iterated integral or integrals equal to the double integral $\displaystyle\iint_R f\,dA$ where $f(x, y) = xy$ and R the region enclosed in the petal of $r = 2\sin(3\theta)$ that lies in the first quadrant. (12 points)

There is more on the flip side.

6. Compute the value of the triple integral $\iiint\limits_D f\,dV$ where $f(x,y,z) = x$ and D is the region in the first octant bounded by the plane $z = 12 - 6x - 3y$. (14 points)

7. Set up an iterated integral or integrals equal to the volume of the solid region that is bounded above by the paraboloid $z = 3 - x^2 - y^2$ and bounded below by the sphere of radius 9 centered at the origin. (12 points)

8. Use spherical coordinates to set up an iterated integral or integrals equal to the triple integral $\iiint\limits_D f\,dV$ where $f(x,y,z) = y$ and D is the portion of the solid sphere $x^2 + y^2 + z^2 \leq 16$ with $y \geq 0$. (12 points)

9. A solid right circular cylinder of radius R and height H has a non-uniform composition so that the volume mass density is proportional to the distance from the central axis of the cylinder.

   (a) Compute the total mass of the cylinder in terms of R, H, and the maxmium density ρ_0 (or δ_0 if you prefer). (10 points)

   (b) Determine if you result in (a) is reasonable by comparing to the total mass of a solid right circular cylinder of radius R and height H having uniform composition with mass density ρ_0 (or δ_0 if you prefer) throughout. (4 points)