Name ______________________________

MATH 280 Multivariate Calculus Fall 2010 Exam #1

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Draw any two vectors u and v that are not equal in magnitude and not perpendicular. Use your two vectors to draw (with reasonable accuracy) each of the following. (4 points each)

   (a) $2\vec{u} + 3\vec{v}$ (b) $2\vec{u} - 3\vec{v}$

2. Let $\vec{u} = \langle 2, -1, 7\rangle$ and $\vec{v} = \langle 9, 5, 2\rangle$. Compute each of the following. (5 points each)

   (a) $4\vec{u} + \vec{v}$ (b) $\vec{u} \cdot \vec{v}$

3. A hot air ballon is rising straight up at a speed of 3 meters per second. The ballon enters an air current moving at 2 meters per second in the direction 30° west of north. Assume the resulting velocity of the ballon is the sum of the original velocity and the air current velocity.

   (a) Determine the new velocity vector of the ballon using a coordinate system with $\hat{\imath}$ pointing east, $\hat{\jmath}$ pointing north, and $\hat{k}$ pointing up. (6 points)

   (b) Determine the new speed of the ballon. (4 points)

4. Find the coordinates of the point that is 4/5 of the way from the point $(2, 1, 7)$ to the point $(-4, 3, 1)$. (8 points)

5. Consider the vectors $\vec{u} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{v} = -2\hat{\imath} - \hat{\jmath}$.

   (a) Make a reasonably accurate sketch showing the vectors $\vec{u}$, $\vec{v}$, and the projection of $\vec{u}$ in the direction of $\vec{v}$ (6 points)

   (b) Compute the projection of $\vec{u}$ in the direction of $\vec{v}$. (6 points)

6. Let $\vec{u}$ and $\vec{v}$ be vectors with $\|\vec{u}\| = 4$, $\|\vec{v}\| = 7$, and $\vec{u} \cdot \vec{v} = -3$.

   (a) Compute the angle between $\vec{u}$ and $\vec{v}$. (6 points)

   (b) Determine the value of $(3\vec{u} - 2\vec{v}) \cdot (\vec{u} + 5\vec{v})$. (6 points)

There is more on the flip side.

7. Find the slopes-intercept form of the equation for the plane that contains the point $(6, 5, 1)$ and has normal vector $4\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$. (9 points)

8. Consider the surface given by the equation $9x^2 - 18y^2 + z^2 = 36$.

   (a) Sketch (with reasonable accuracy) the cross-sections of this surface in the xy-plane, the xz-plane, and the yz-plane. (9 points)

   (b) Use words and/or pictures to describe the surface. You can include additional cross-sections for this if useful. (6 points)

9. Do any two of the following four problems. Circle the letters for the two problems you submit. (8 points each)

   (A) Show that the plane given by the equation $-6x + 4y - 14z = 1$ is parallel to the plane given by the equation $z = -\frac{3}{7}x + \frac{2}{7}y + 3$.

   (B) Show that it is *not* possible to have vectors $\vec{u}$ and $\vec{v}$ with $\|\vec{u}\| = 2$, $\|\vec{v}\| = 3$, and $\vec{u} \cdot \vec{v} = -8$.

   (C) Let $\vec{u} = \langle 2, 1, 0 \rangle$, $\vec{v} = \langle 1, 5, 3 \rangle$, and $\vec{w} = \langle -4, 7, 6 \rangle$. Find numbers a and b so that $\vec{w} = a\vec{u} + b\vec{v}$.

   (D) Find the center and radius of the sphere given by the equation $x^2 + y^2 + z^2 + 3x - 5y = 0$.