# Project #3

**Instructions:** Refer to the earlier handout on general project requirements for details on expectations. One way to approach your writing is to think of authoring an example in a textbook. Your report should be self-contained so that a reader with a background in multivariate calculus can follow your reasoning and results. As part of this, you should introduce the context by either paraphrasing the background given below or developing your own introduction.

The project is due on Monday, May 2.

**Background:** A hydrogen atom consists of one proton and one electron. A *free* hydrogen atom is one that experiences no external forces. In a free hydrogen atom, the electron can be in one of infinitely many discrete states. These states are labeled by three integers, usually denoted $n$, $l$, and $m$. For each state, there is an *electron location probability density* that gives the volume probability density for the location of the electron as a function of position (measured with respect to the proton).

The $n = 3$, $l = 2$, $m = 0$ state of a free hydrogen atom has an electron probability volume density given by

$$\delta(\rho, \phi, \theta) = \frac{1}{39366\pi} \rho^4 e^{-2\rho/3} (3\cos^2\phi - 1)^2$$

where $(\rho, \phi, \theta)$ are spherical coordinates as we use them in class. The origin of the coordinate system is the location of the proton. The radial coordinate $\rho$ is measured in units of *Bohr radii* where the Bohr radius is equal to about $5.3 \times 10^{-11}$ meters. (So, for example, $\rho = 2$ means a radial distance of 2 Bohr radii.)

**Objective:** For this project, your objective is to get insight on this specific probablility density function by computing some total probabilities. Below are some calculations and questions you could pursue in order to get some insight. (Note that a result from the first suggestion can be used for the other two suggestions). You are free to augment these or to take a different approach. In whatever approach you take, you should compute some total probabilities.

- Compute an expression for the probability of finding the electron between $\rho = a$ and $\rho = b$ for a hydrogen atom in this state.
- Compute the probability of finding the electron anywhere in space. Does this result make sense?
- Compute the probability of finding the electron in each of the spherical shells $\rho = k$ to $\rho = k + 1$ for $k$ between 0 and 25. (In other words, compute the probability of finding the electron between $\rho = 0$ and $\rho = 1$, between $\rho = 1$ and $\rho = 2$, and so on.) In which of these intervals is the electron most likely to be found?