Project #1

For this project, you have a choice of two options. In each option, the main idea is to use mathematical tools from this course to get some insight on particular functions of two or more variables that arise in applications. Your project report should allow readers to follow your analysis and understand your insights.

You should focus on doing your own mathematical analysis rather than looking for resources in which these functions are described or analyzed. You can focus on comparing the mathematical features of the given functions. You are welcome to also think about what those features mean in a real-world context. If you do use ideas or material from resources other than our textbook, you must give a proper citation.

The project is due on Friday, March 4.

Option 1

Background In class, we have used the *ideal gas law* as an example. The context for the ideal gas law is having some amount of gas in a situation where we can control or measure various quantites such as the number of gas particles n (typically in mols), pressure p (typically in pascals or atmospheres), volume V (typically in m^3 or liters), and temperature T (in Kelvin). For a gas modeled as ideal, these are related by $pV = nRT$ where R is the *universal gas constant* having value $R = 8.314$ J/(mol·K)=0.082 L·atm/(mol·K).

Not all gases are modeled well by the ideal gas law. The *van der Waals law* is a generalization of the idea gas law that accounts for two factors:

- an attractive force between any pair of gas particles, and
- volume occupied by the gas particles themselves.

The van der Waals formula is

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT.$$

Note that the van der Waals law formula includes two parameters a and b. These are related to the two factors. Values for these parameters depend on the type of gas. As example, for carbon dioxide (CO_2) we have $a = 3.59$ L^2·atm/mol^2 and $b = 0.043$ L/mol.

If we consider a situation in which the gas does not change (so a and b are constant) and the number of particles does not change (so n is constant), then we have three variables: p, V, and T. We can think of one of these as a function of the other two. In particular, we will consider T as a function of p and V.

Objective For this project, your task is to use mathematical ideas from this course to compare the ideal gas law with the van der Waals gas law. In particular, you should find and describe at least one mathematical similarity and at least one mathematical distinction between the two laws. Strive for finding interesting or relevant comparisons. You can assume readers in your audience are familiar with the ideal gas law but are not familiar with the van der Waals gas law.

Option 2

Background In class, we looked at the *Cobb-Douglas model* as an example of a *consumer utility function.* Consumer utility functions arise in the context of studying how a consumer makes choices about purchasing commodities. In a simplified situation, we think of the consumer as purchasing a *bundle* consisting of certain amounts of different commodities. *Utility* is an abstract measure of the value of each bundle to the consumer.

Here, we will limit attention to having a commodity bundle with just two types of thing. Let x and y denote the amount of each thing in a bundle. For convenience, we can think of x and y as measuring weight in pounds (lb). We'll use the made-up unit *util* for the unit for utility U.

The Cobb-Douglas utility function is

$$U = x^p y^{1-p}$$

for some constant p with $0 < p < 1$. (Note that the exponents on x and y sum to 1.) Another commonly used example is a *constant elasticity of substitution (CES)* utility function. The CES utility function involves two parameters a and b in the form

$$U = [ax^b + (1-a)y^b]^{1/b}.$$

For a, we use $0 < a < 1$ while b can have any positive value.

Objective For this project, your task is to use mathematical ideas from this course to compare the Cobb-Douglas utility function with the CES utility function. In particular, you should find at least one mathematical similarity and at least one mathematical distinction between the two utility functions. Strive for finding interesting or relevant comparisons. You can assume readers in your audience are familiar with the Cobb-Douglas utility function but are not familiar with the CES utility function.