Continuity and limit

Math 280

Spring 2011

Continuity

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Need precise definition of limit

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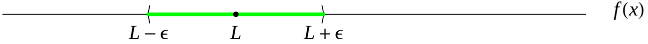

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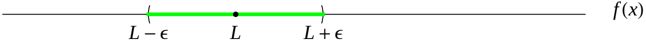

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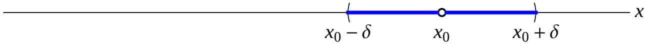

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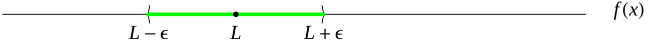

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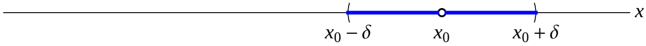

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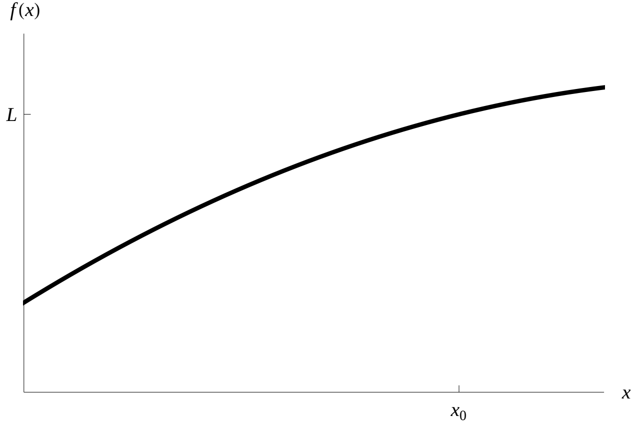

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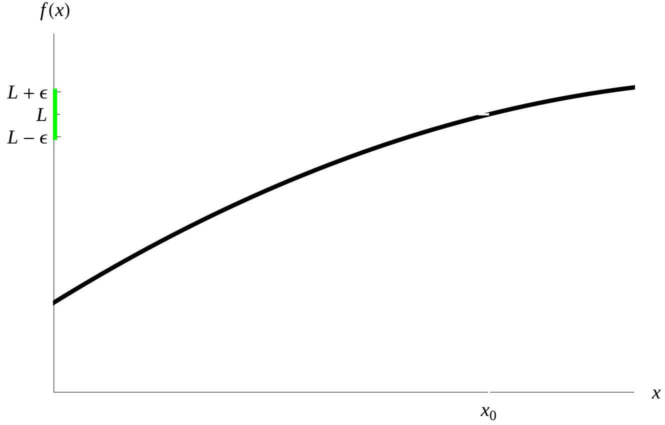

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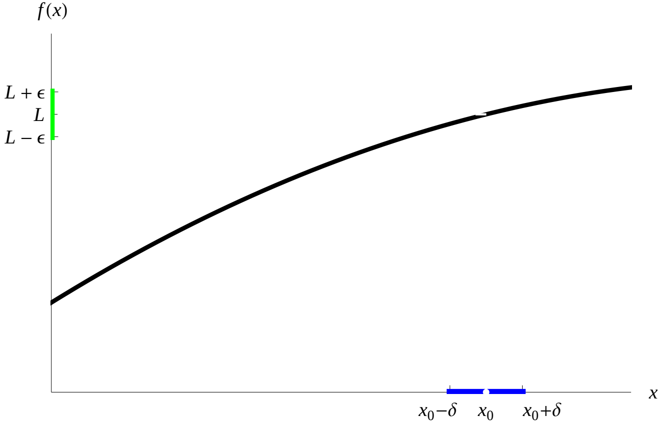

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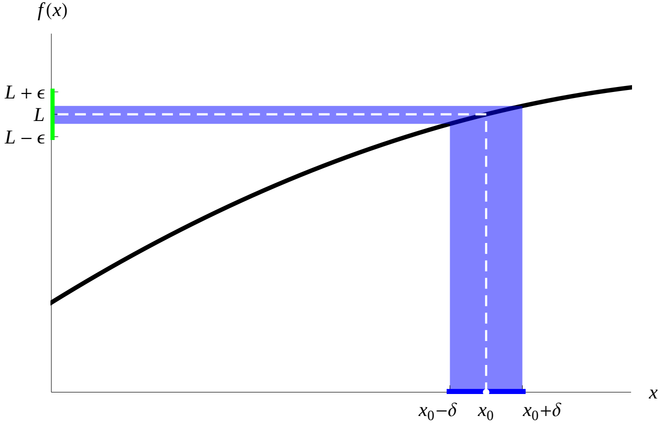

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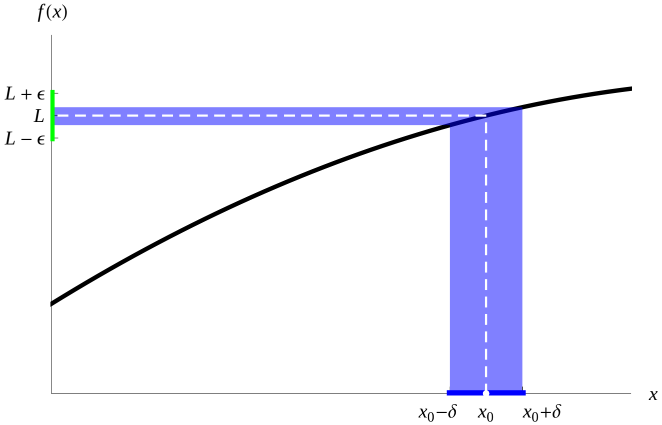

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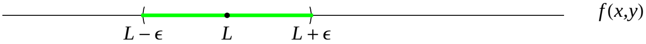

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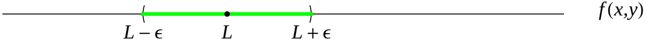

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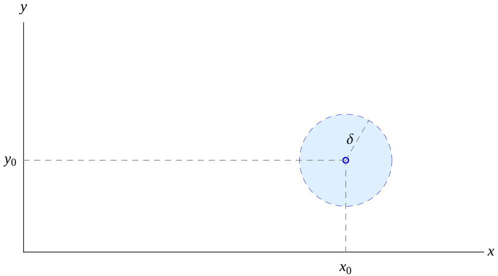

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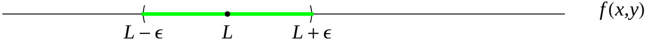

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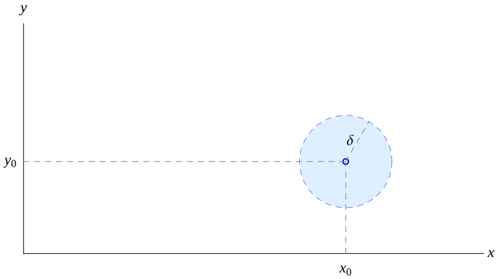

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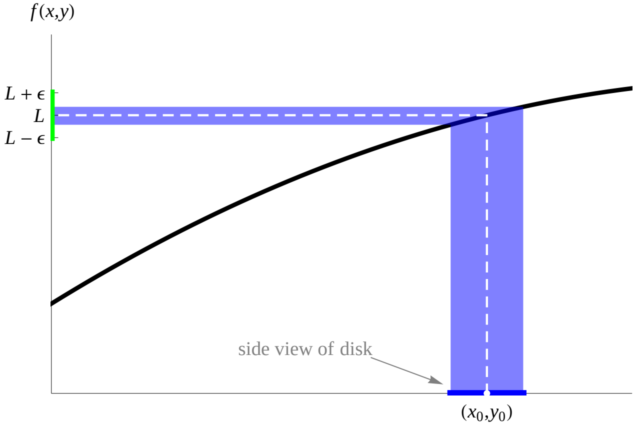

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