Continuity and limit

Math 280

Spring 2011

Continuity

For $f : \mathbb{R} \to \mathbb{R}$

- "unbroken curve"
- $\lim_{x \to x_0} f(x)$ exists and equal to $f(x_0)$

For $f : \mathbb{R}^2 \to \mathbb{R}$

- "unbroken surface"
- $\lim_{(x,y) \to (x_0,y_0)} f(x, y)$ exists and equal to $f(x_0, y_0)$

For $f : \mathbb{R}^3 \to \mathbb{R}$

- "unbroken ???"
- $\lim_{(x,y,z) \to (x_0,y_0,z_0)} f(x, y, z)$ exists and equal to $f(x_0, y_0, z_0)$

Need precise definition of limit

Limits for $f : \mathbb{R} \to \mathbb{R}$

Distinguish between

- What is it?
- How do we compute/evaluate it?

Example: Evaluate $\lim_{x \to 0} \frac{\sin(x)}{x}$

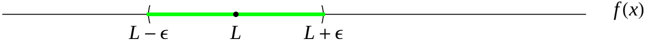
- conjecture based on table of values
- compute using L'Hopital's rule

What does it mean to say $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$?

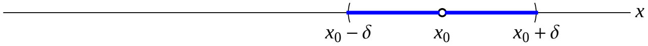
A precise definition

Definition: L is the limit of f at x_0 if for every **target** centered at L, there is a **successful launch pad** centered at x_0.

Target: an open interval centered at L



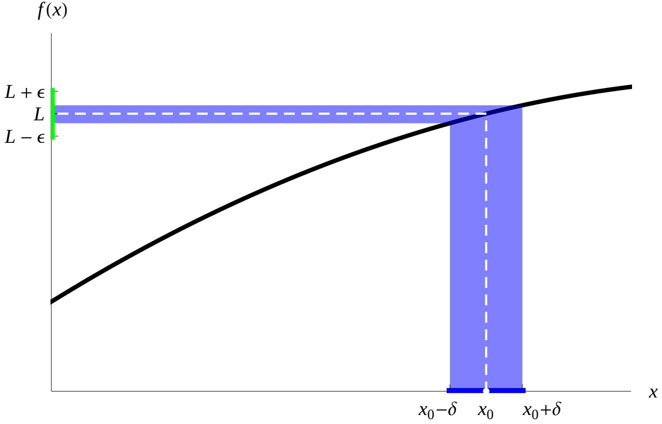
Launch pad: an open interval centered at x_0 with x_0 taken out



Successful launch pad: every input x in the launch pad has an output $f(x)$ in the target

A precise definition

Definition: L is the limit of f at x_0 if for every **target** centered at L, there is a **successful launch pad** centered at x_0.



Traditional phrasing of a precise definition

Brief	**Verbose**	**Traditional**
For each target centered at L	For each open interval centered at L	For each $\epsilon > 0$
there is a launch pad centered at x_0	there is an open interval centered at x_0 with x_0 removed	there is a corresponding number $\delta > 0$
that is successful.	such that x in the launch pad has $f(x)$ in the target.	such that $0 < \lvert x - x_0 \rvert < \delta$ implies $\lvert f(x) - L \rvert < \epsilon$.

Definition: L is the limit of f at x_0 if for every $\epsilon > 0$, there is a corresponding $\delta > 0$ such that $0 < |x - x_0| < \delta$ implies $|f(x) - L| < \epsilon$.

A precise definition

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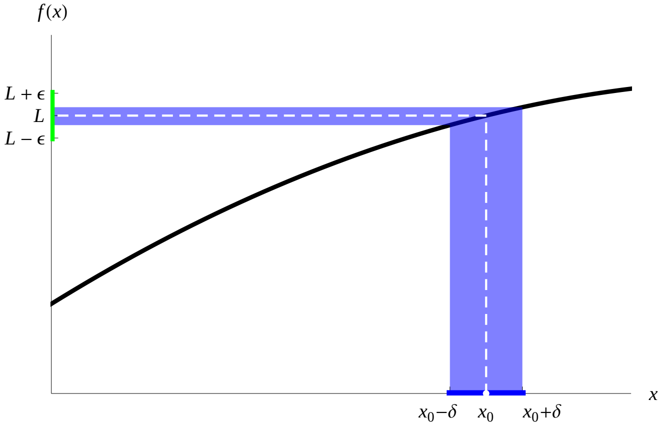
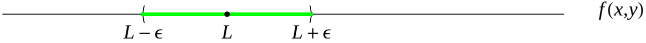


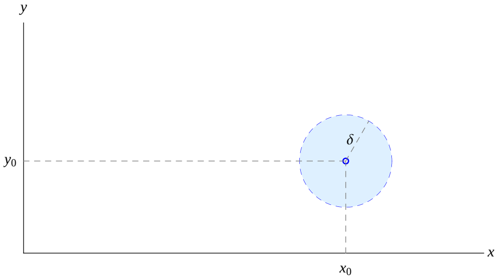
Limits for $f : \mathbb{R}^2 \to \mathbb{R}$

Definition: L is the limit of f at (x_0, y_0) if for every **target** centered at L, there is a **successful launch pad** centered at (x_0, y_0).

Target: an open interval centered at L



Launch pad: an open disk centered at (x_0, y_0) with (x_0, y_0) taken out



Successful launch pad: every input (x, y) in the launch pad has an output $f(x, y)$ in the target

Limits for $f : \mathbb{R}^2 \to \mathbb{R}$

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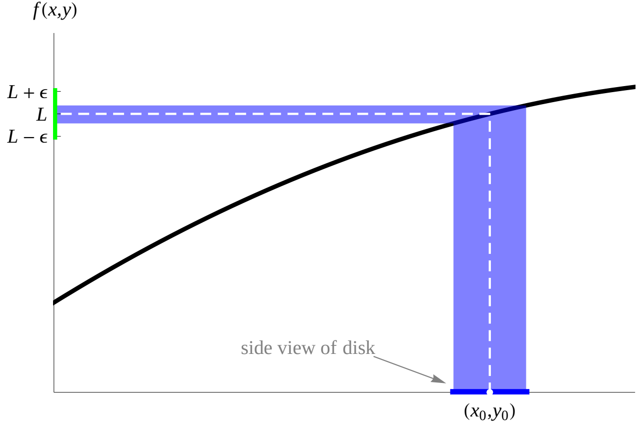

Limits for $f : \mathbb{R}^2 \to \mathbb{R}$

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