Estimating greatest rate of change

The accompanying plot shows level curves for a function $f : \mathbb{R}^2 \to \mathbb{R}$. We can think of each input as a point on a plane and the corresponding output as a temperature. We will consider distance to be measured in kilometers (km) and temperature to be measured in degrees Celsius (°C). There is a scale for distance at the bottom of the plot. A selection of level curves is labeled with the corresponding temperature.

1. For the point A, estimate the direction of the greatest rate of change in temperature with respect to change in position.

2. For the point A, estimate the magnitude of this greatest rate of change.

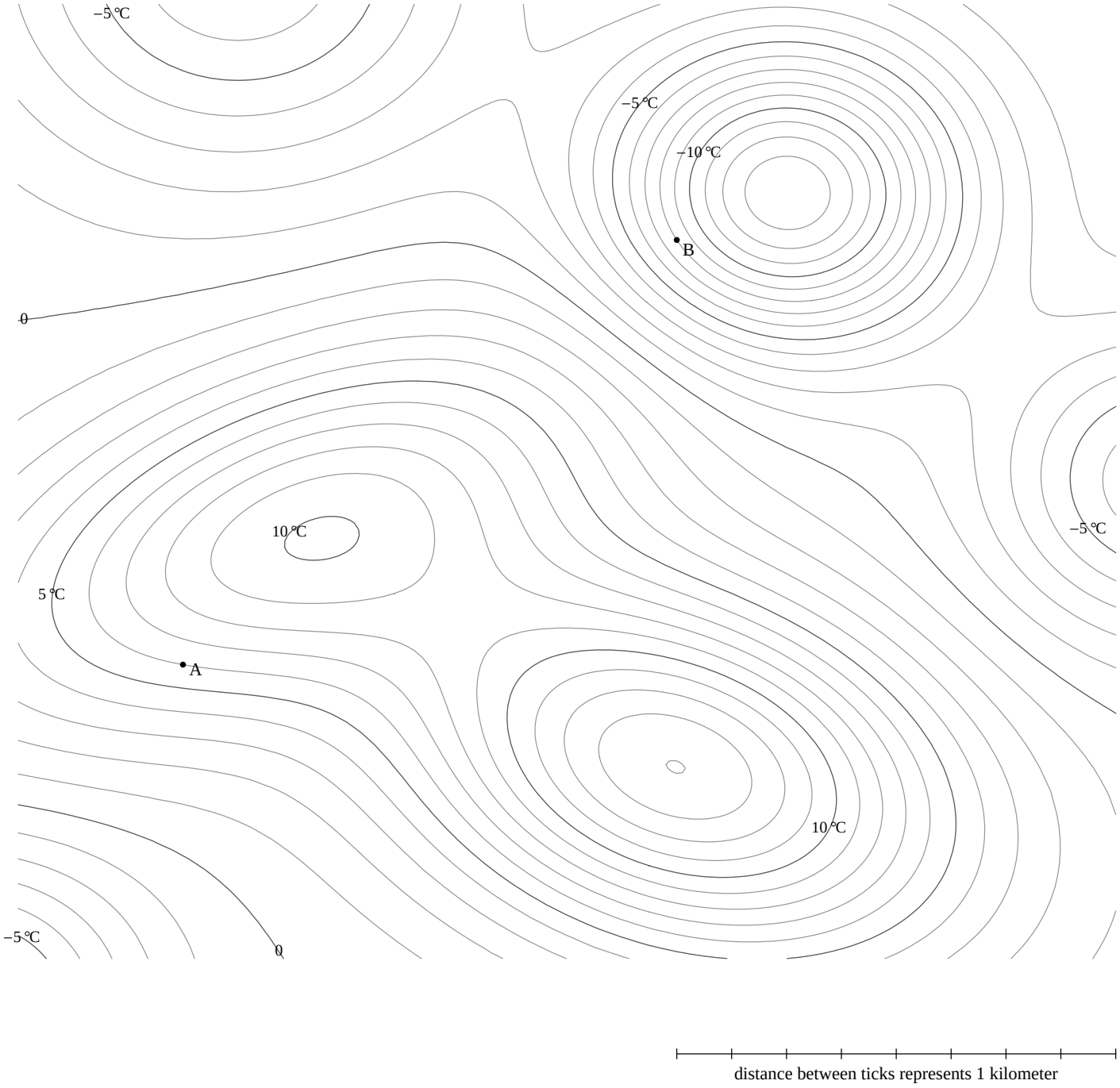
3. Choose a scale for rate of change. Note that this scale is independent of the scale for distance. With the temperature interpretation, rate of change has units of degrees Celsius per kilometer (°C/km) while the length scale is in kilometers (km). To choose a scale for rate of change, go to the bottom of the plot next to the given length scale and draw a horizontal vector (of any size you want) to represent a magnitude of 1 °C/km. You will use this choice in what follows.

4. At the point A, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.

5. For the point B, estimate the direction of the greatest rate of change in temperature with respect to changes in position.

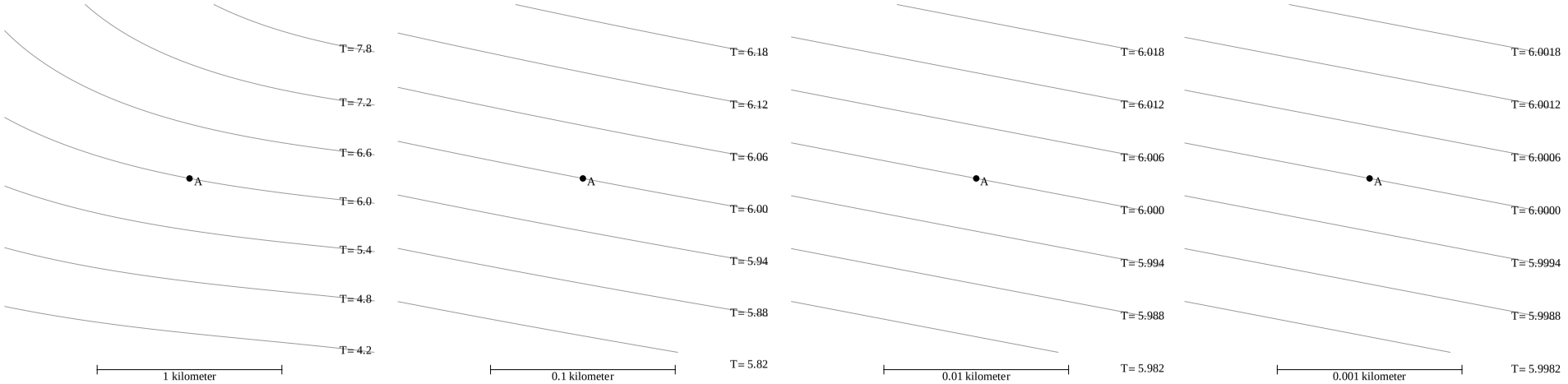
6. For the point B, estimate the magnitude of this greatest rate of change.

7. At the point B, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.



Level curves for temperature as a function of position.

Zooming in on Point A



Zooming in on Point B