Spring 2011

Computing directional derivatives

Suppose we have a function $f : \mathbb{R}^2 \to \mathbb{R}$ and a point P in the domain of f. For the infinitesimal displacement $d\vec{r}$, the corresponding change in f is given by

$$df = \vec{\nabla} f \cdot d\vec{r} \tag{1}$$

which we can think of as a rise. The corresponding run is given by the magnitude of $d\vec{r}$. For notational convenience, we will denote this magnitude as $ds = ||d\vec{r}||$.

- 1. Draw a picture showing
 - a point P,
 - the level curve (labeled f_0) passing through P,
 - a displacement $d\vec{r}$ at P that is not perpendicular to the level curve through P,
 - the level curve labeled $f_0 + df$, and
 - the gradient vector $\vec{\nabla} f$ at P.

- 2. Divide both sides of Equation (1) by ds.
- 3. What does the left side of the equation in Step 2 represent?
- 4. On the right side of the equation you get from Step 2, you should be able group some of the factors to form a unit vector. Name this unit vector \hat{u} . Rewrite the equation using \hat{u} . Include \hat{u} in your picture from Step 1.
- 5. Complete the following sentence that describes one way to read the equation in Step 4:

To compute the rate of change in f in the direction \hat{u} ,

6. For the point (1,2), compute the rate of change in $f(x,y) = x^2 y^3$ in the direction $\hat{u} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$. Include a rough sketch of the relevant point and vectors.

Answer:
$$\frac{df}{ds} = 20$$

7. For the point (-2, 4), compute the rate of change in $f(x, y) = x\sqrt{y}$ in the direction of the vector $\vec{v} = \hat{i} - 3\hat{j}$. Include a rough sketch of the relevant point and vectors.

Answer:
$$\frac{df}{ds} = \frac{7}{2\sqrt{10}}$$

8. For the point (1,2), compute the rate of change in $f(x,y) = y \sin(\pi x)$ in the direction of the point (3,7). Include a rough sketch of the relevant points and vectors.

Answer: $\frac{df}{ds} = \frac{-4\pi}{\sqrt{29}}$