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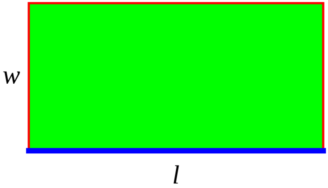

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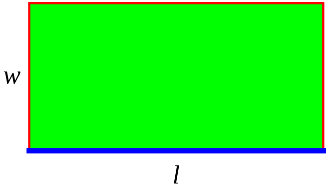

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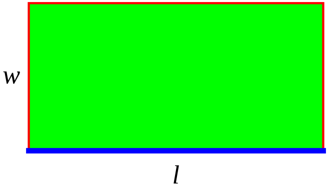

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Constraint: Need $lw = 1200$.

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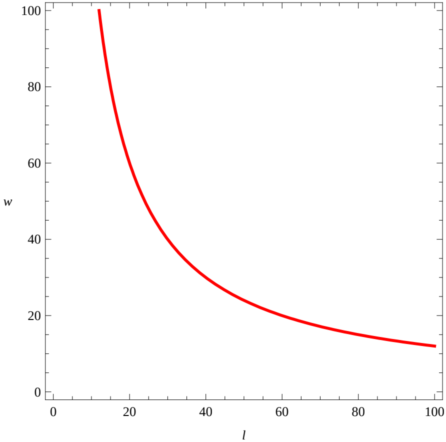
So build fence with expensive edge of length 30 meters and other dimension of 40 meters.

Idea for Method 2



Constraint curve $A = lw = 1200$

Level curves for objective $C = 80l + 60w$

Gradient vectors for constraint $A = lw$

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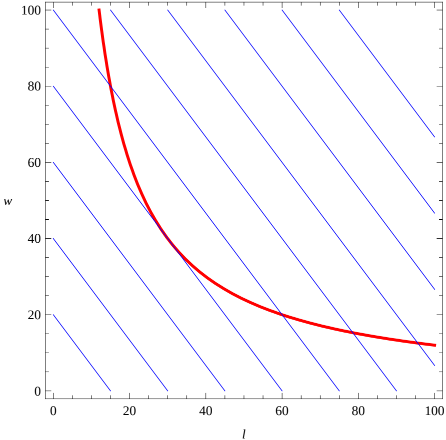

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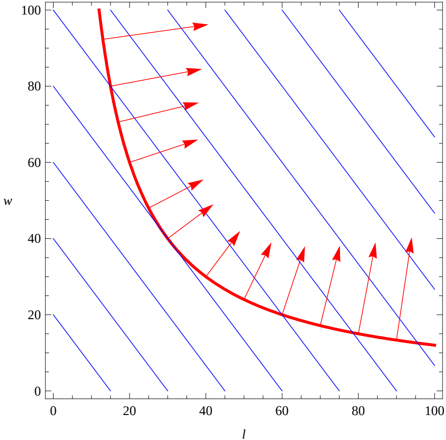

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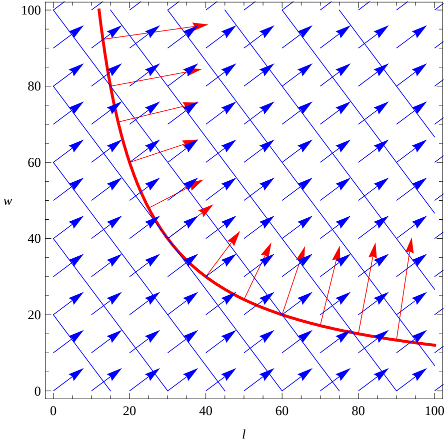

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$\vec{\nabla} C = \lambda \vec{\nabla} A$ for some constant λ