Clairaut's Theorem

Math 280

Spring 2011

Nature of mathematics

For many mathematicians, main focus is *not* on applying mathematics.

Focus is on determining **true** mathematical statements.

True statements are either

- given by definition or axiom; or
- proven by logic using definitions, axioms, and previously proved statement

Proven statements are phrased as **theorems**, often in the form

If *hypotheses*, then *conclusion*

Hypotheses give precise conditions under which the conclusion is guaranteed to hold.

Equality of mixed partials

Have seen several examples in which

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \qquad \text{or} \qquad f_{xy} = f_{yx}$$

There are functions for which this is not true.

Example:

$$\text{For } f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

have $f_{xy}(0,0) = -1$ while $f_{yx}(0,0) = 1$.

Theorem:
If (x_0, y_0) is a point in the domain of a function f with

(A) f defined for all points in an open disk centered at (x_0, y_0); and

(B) f_x, f_y, f_{xy}, and f_{yx} each continuous for all points in that open disk

then $f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$.